Design against Static Loads

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A static load is defined as a force, which is gradually applied to a mechanical component and which does not change its magnitude or direction with respect to time.

Modes of failure

- **failure by elastic deflection**
- **failure by general yielding**
- **failure by fracture**

FACTOR OF SAFETY: **While designing a component, it is necessary to provide sufficient reserve strength in case of an accident. This is achieved by taking a suitable factor of safety (*fs*). The factor of safety is defined as**

$$(fs) = \frac{\text{failure stress}}{\text{allowable stress}} \qquad (fs) = \frac{\text{failure load}}{\text{working load}}$$

The *allowable stress* is the stress value, which is used in design to determine the dimensions of the component. It is considered as a stress, which the designer expects will not be exceeded under normal operating conditions

For ductile materials, the allowable stress is obtained by the following relationship: $\sigma = \dfrac{S_{yt}}{(fs)}$

For brittle materials, the relationship is, $\sigma = \dfrac{S_{ut}}{(fs)}$

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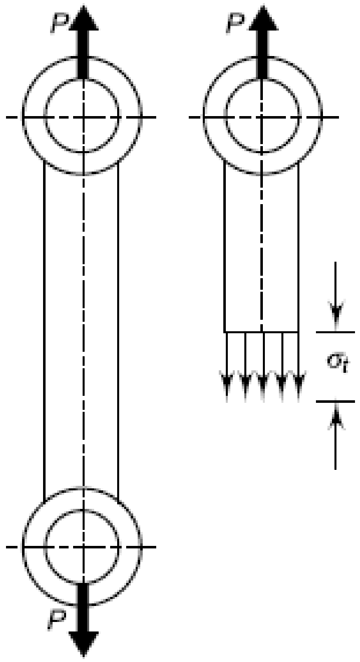
STRESS–STRAIN RELATIONSHIP: **When a mechanical component is subjected to an external static force, a resisting force is set up within the component. *The internal resisting force per unit area of the component is called stress*.**

- **The stresses are called *tensile* when the fibres of the component tend to elongate due to the external force.**
- **On the other hand, when the fibres tend to shorten due to the external force, the stresses are called *compressive stresses*.**

Tensile Stress

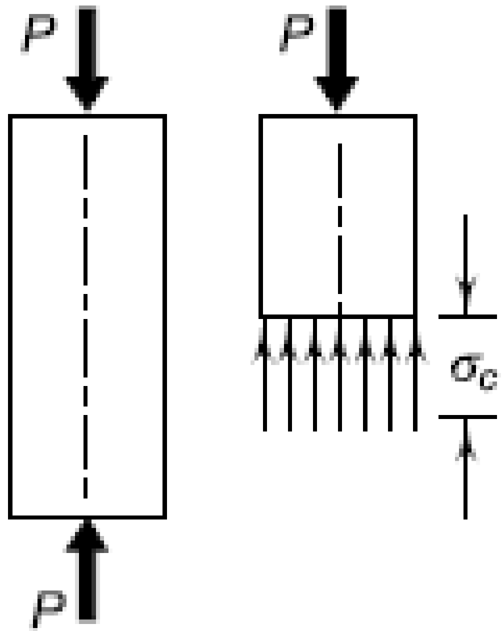


$$\sigma_t = \frac{P}{A}$$

where

σ_t = tensile stress (N/mm^2)
P = external force (N)
A = cross-sectional area (mm^2)

Compressive Stress



$$\sigma_c = \frac{P}{A}$$

*The **strain** is deformation per unit length*. **It given by**

$$\boxed{\varepsilon = \frac{\delta}{l}}$$

where,

ε = strain (mm/mm)

δ = elongation of the tension rod (mm)

l = original length of the rod (mm)

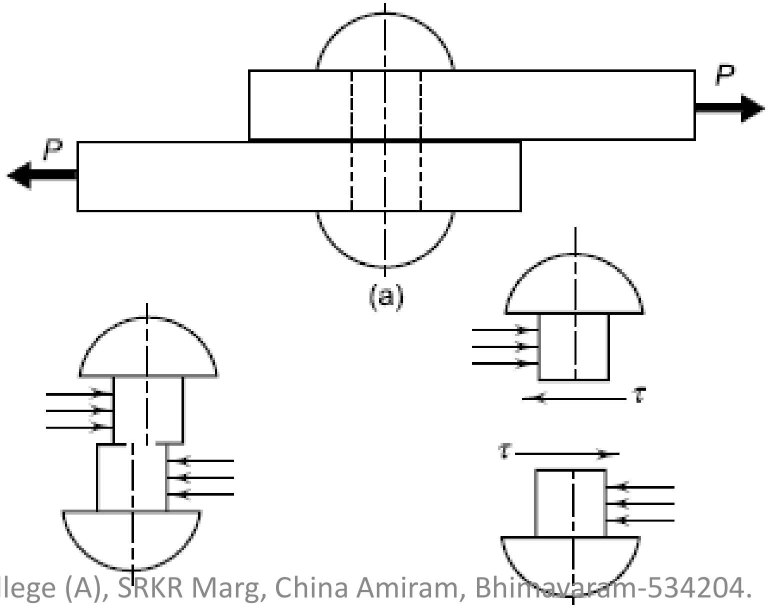
According to Hooke's law, the stress is directly proportional to the strain within elastic limit. Therefore,

$$\boxed{\begin{gathered}\sigma_t \,\alpha\, \varepsilon \\ \sigma_t = E\,\varepsilon\end{gathered}}$$

where *E* is the constant of proportionality known as *Young's modulus* or *modulus of elasticity* (in N/mm^2 or MPa).

***SHEAR STRESS AND SHEAR STRAIN:* *When the external force acting on a component tends to slide the adjacent planes with respect to each other, the resulting stresses on these planes are called direct shear stresses*.**

$$\boxed{\tau = \frac{P}{A}}$$



where,

τ = shear stress (N/mm^2 or MPa)

A = cross-sectional area of the rivet (mm^2)

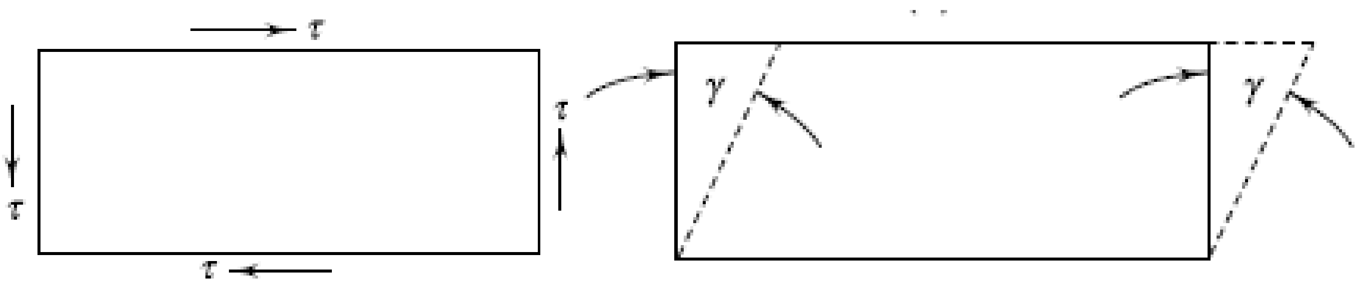
The *shear strain* *is defined as the change in the right angle of a shear element*. **Within the elastic limit, the stress–strain relationship is given by**

$$\boxed{\tau = G\gamma}$$

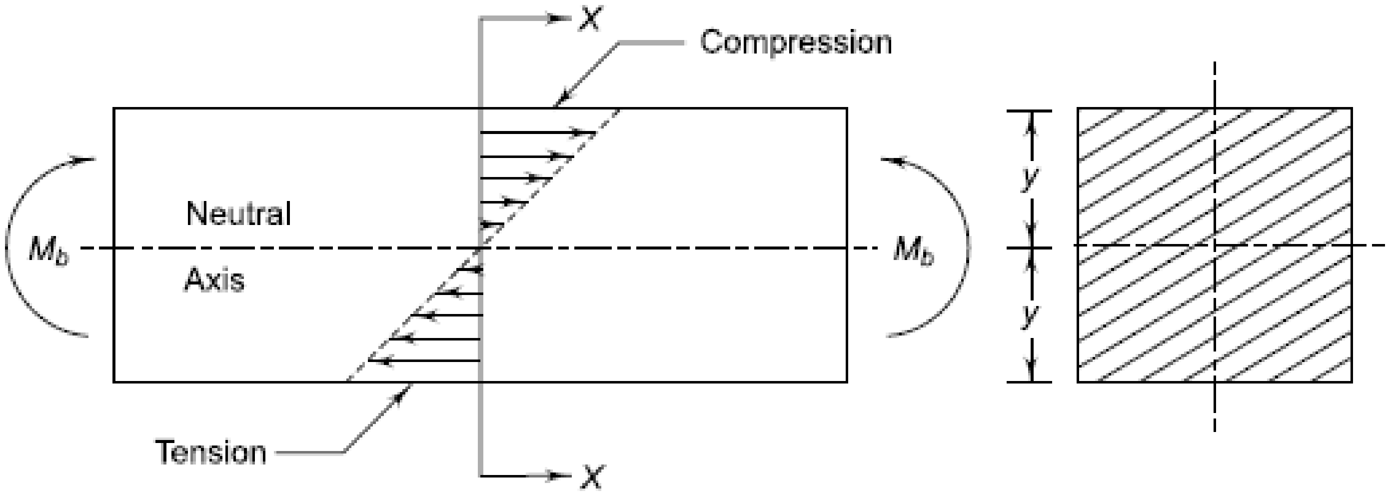
where,

γ = shear strain (radians)

G is the constant of proportionality known as *shear modulus* or *modulus of rigidity* (in N/mm^2 or MPa).



STRESSES DUE TO BENDING MOMENT: **A straight beam subjected to a bending moment *Mb* is shown in Figure below. The beam is subjected to a combination of tensile stress on one side of the neutral axis and compressive stress on the other. The bending stress at any fibre is given by:**



$$\boxed{\sigma_b = \frac{M_b\, y}{I}}$$

where,

σ_b = bending stress at a distance of y from the neutral axis (N/mm^2 or MPa)

M_b = applied bending moment (N-mm)

I = moment of inertia of the cross-section about the neutral axis (mm^4)

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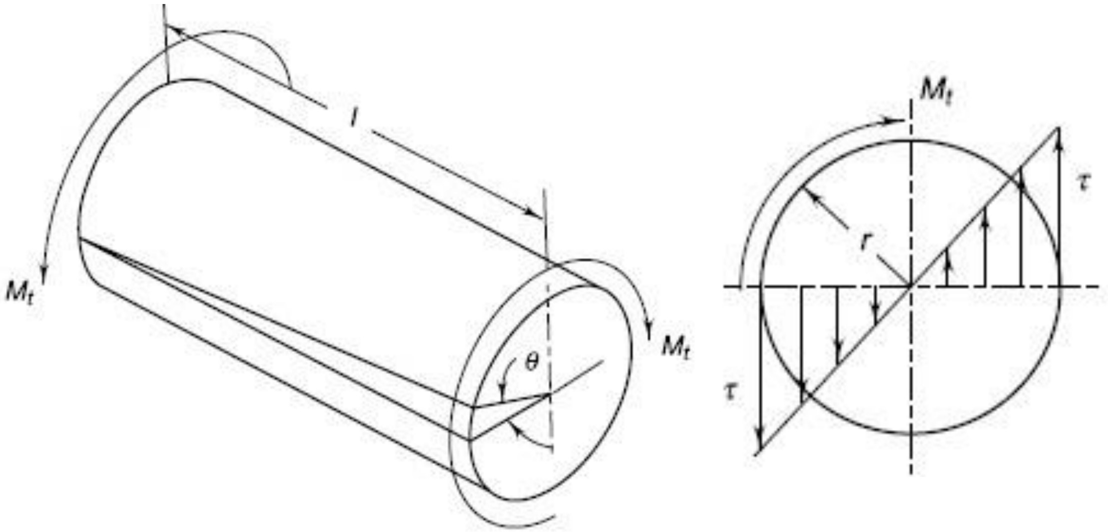
STRESSES DUE TO TORSIONAL MOMENT: **A transmission shaft, subjected to an external torque, is shown in Figure below.** ***The internal stresses, which are induced to resist the action of twist, are called torsional shear stresses*****. The torsional shear stress is given by:**

$$\tau = \frac{M_t\, r}{J}$$



where,

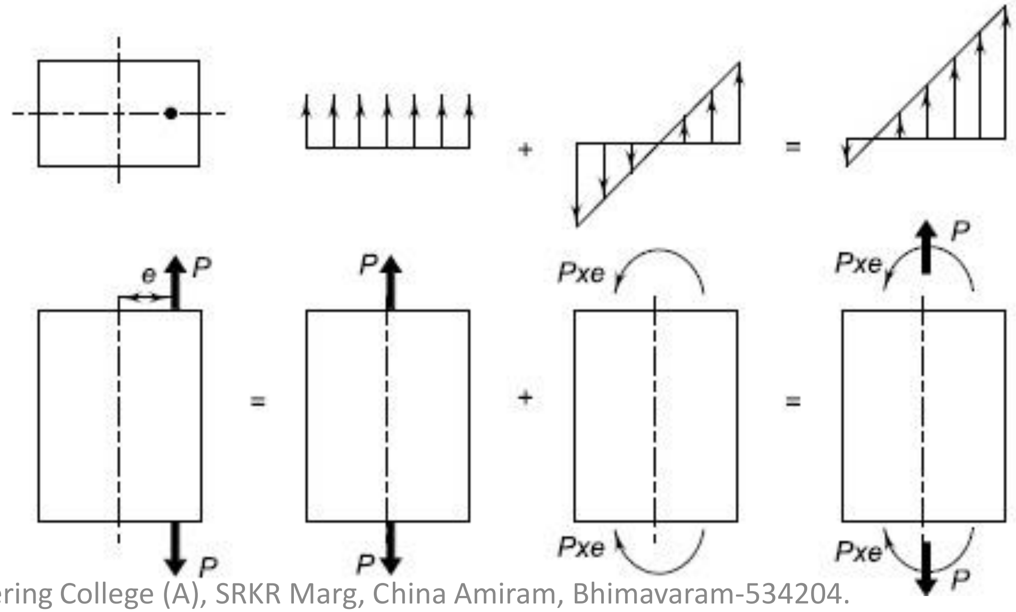
τ = torsional shear stress at the fibre (N/mm^2 or MPa)

M_t = applied torque (N-mm)

r = radial distance of the fibre from the axis of rotation (mm)

J = polar moment of inertia of the cross-section about the axis of rotation (mm^4)

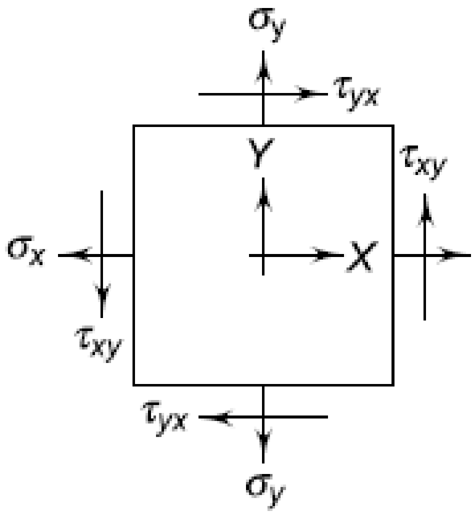
ECCENTRIC AXIAL LOADING: **There are certain mechanical components subjected to an external force, tensile or compressive, which does not pass through the centroid of the cross-section. A typical example of such an eccentric loading is shown in Figure aside. The resultant stresses at the cross-section are obtained by the principle of superimposition of stresses. They are given by,**

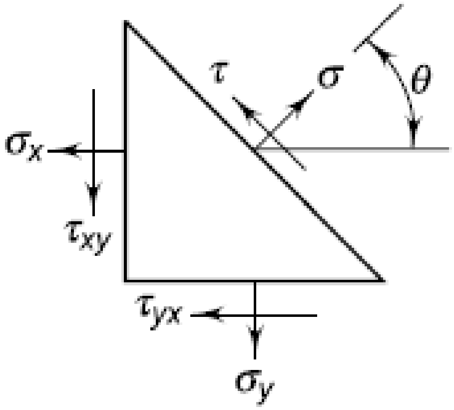


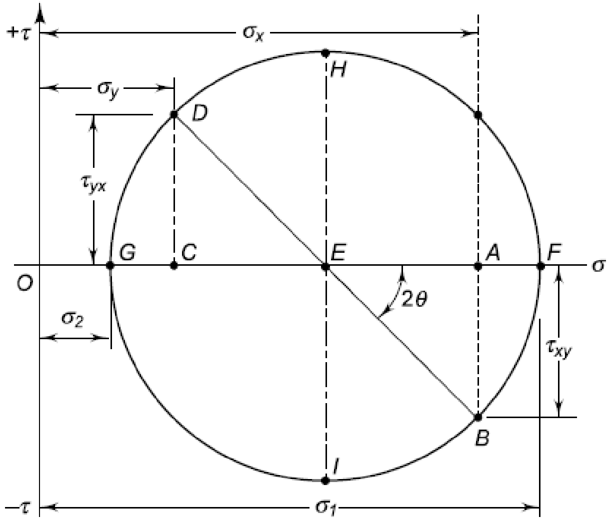
$$\sigma = \frac{P}{A} \pm \frac{Pey}{I}$$

Static Failure Theories

Principal Stresses







$$\sigma = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta + \tau_{xy}\sin 2\theta \quad \textbf{Eq. (1)}$$

$$\boxed{\sigma_1 = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + (\tau_{xy})^2}} \quad \textbf{Eq. (5)}$$

$$\tau = -\left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta \quad \textbf{Eq. (2)}$$

$$\boxed{\sigma_2 = \left(\frac{\sigma_x + \sigma_y}{2}\right) - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + (\tau_{xy})^2}} \quad \textbf{Eq. (6)}$$

$$\tan 2\theta = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} \quad \textbf{Eq. (3)}$$

$$\boxed{\tau_{max.} = \pm\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + (\tau_{xy})^2}} \quad \textbf{Eq. (7)}$$

$$\tan 2\theta = -\left(\frac{\sigma_x - \sigma_y}{2\tau_{xy}}\right) \quad \textbf{Eq. (4)}$$

Theories of Elastic Failure

- There are number of machine components, which are subjected to several types of loads simultaneously.

- For example, a power screw is subjected to torsional moment as well as axial force. Similarly, an overhang crank is subjected to combined bending and torsional moments. The bolts of the bracket are subjected to forces that cause tensile stress and shear stress. Crankshaft, propeller shaft and connecting rod are examples of components subjected to complex loads.

- When the component is subjected to several types of loads, combined stresses are induced. For example, torsional moment induces torsional shear stress, while bending moment causes bending stresses in the transmission shaft.

- **The design of machine parts subjected to combined loads should be related to experimentally determined properties of material under ‘similar’ conditions.**
- **However, it is not possible to conduct such tests for different possible combinations of loads and obtain mechanical properties.**
- **In practice, the mechanical properties are obtained from a simple tension test. They include yield strength, ultimate tensile strength and percentage elongation. In the tension test, the specimen is axially loaded in tension. It is not subjected to either bending moment or torsional moment or a combination of loads.**
- ***Theories of elastic failure provide a relationship between the strength of machine component subjected to complex state of stresses with the mechanical properties obtained in tension test.***
- **With the help of these theories, the data obtained in the tension test can be used to determine the dimensions of the component, irrespective of the nature of stresses induced in the component due to complex loads.**

Theories of Elastic Failure

The principal theories of elastic failure are as follows:

1. **Maximum principal (or normal) stress theory (also known as Rankine's theory).**
2. **Maximum shear stress theory (also known as Guest's or Tresca's theory).**
3. **Maximum principal (or normal) strain theory (also known as Saint Venant theory).**
4. **Maximum strain energy theory (also known as Haigh's theory).**
5. **Maximum distortion energy theory (also known as Hencky and Von Mises theory).**

Maximum Principal Stress Theory

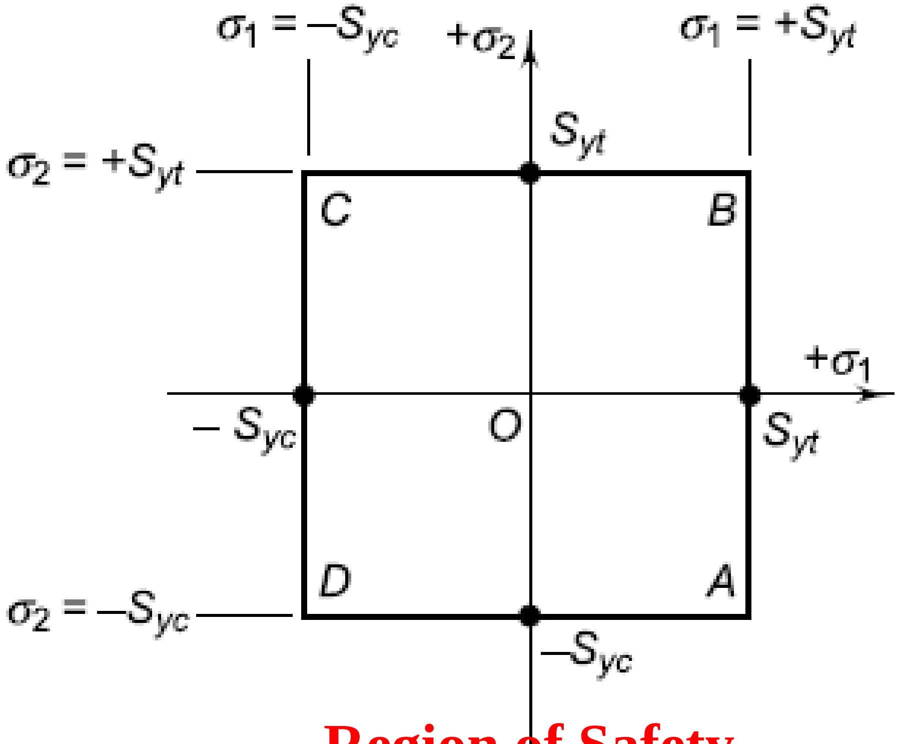
The theory states that the failure of the mechanical component subjected to bi-axial or tri-axial stresses occurs when the maximum principal stress reaches the yield or ultimate strength of the material.

For tensile stresses,

$$\sigma_1 = \frac{S_{yt}}{(fs)} \quad \text{or} \quad \sigma_1 = \frac{S_{ut}}{(fs)}$$

For compressive stresses,

$$\sigma_1 = \frac{S_{yc}}{(fs)} \quad \text{or} \quad \sigma_1 = \frac{S_{uc}}{(fs)}$$



Region of Safety

Maximum Shear Stress Theory

The theory states that the failure of a mechanical component subjected to bi-axial stresses occurs when the maximum shear stress at any point in the component becomes equal to the maximum shear stress in the standard specimen of the tension test, when yielding starts.

$$\tau_{max} = \tau_{yt} / fs$$

(or)

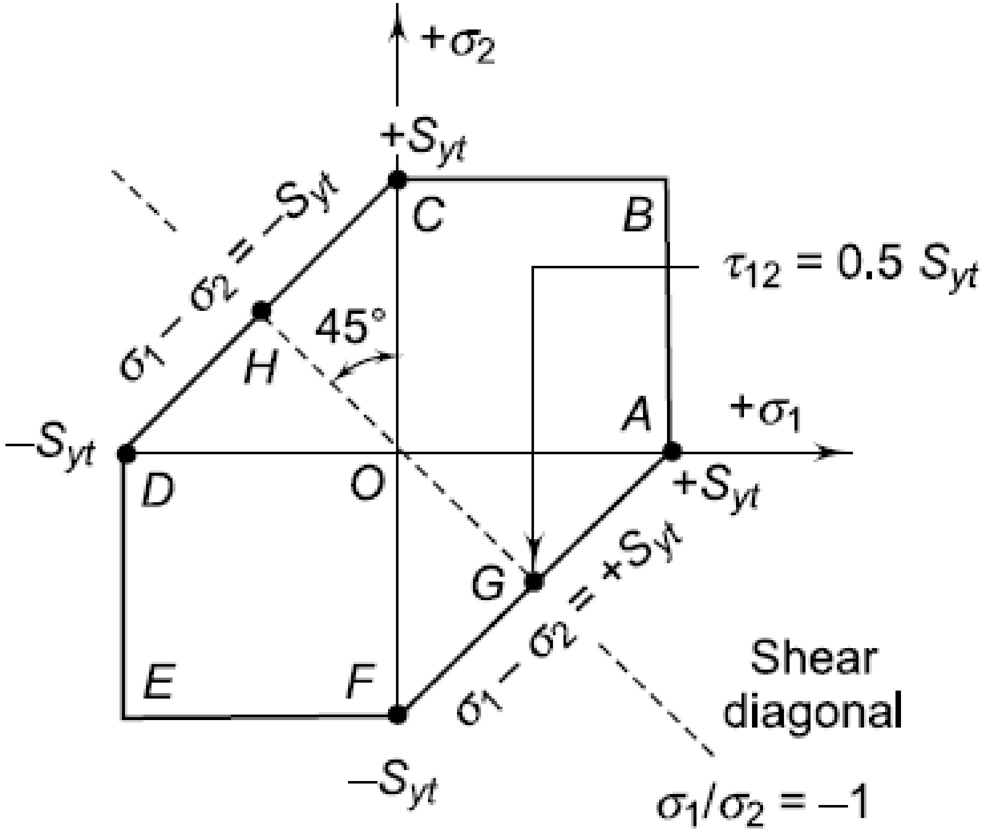
$$\tau_{max} = S_{yt} / (fs \times 2)$$

(or)

$$(\sigma_1 - \sigma_2)/2 = S_{yt}/(fs \times 2)$$

(or)

$$\boxed{\sigma_1 - \sigma_2 = \frac{S_{yt}}{(fs)}}$$

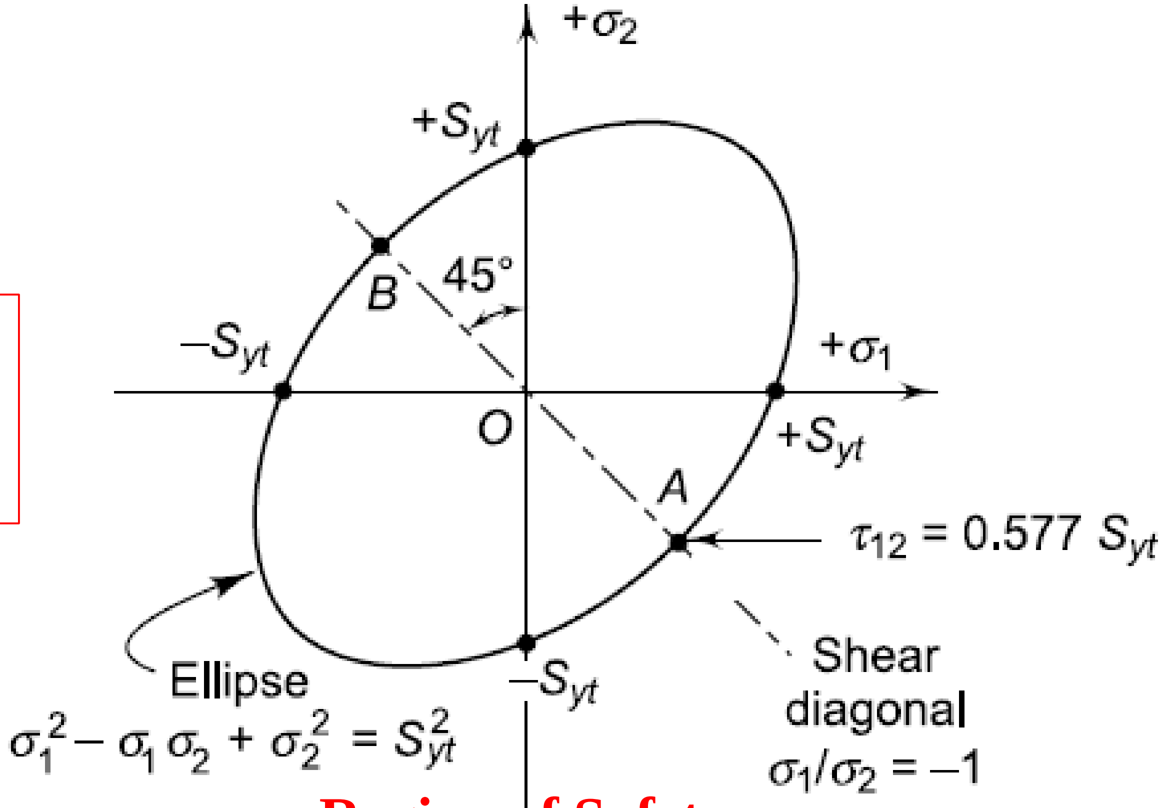


Region of Safety

Maximum Distortion Energy Theory

The theory states that the failure of the mechanical component subjected to bi-axial stresses occurs when the strain energy of distortion per unit volume at any point in the component, becomes equal to the strain energy of distortion per unit volume in the standard specimen of tension-test, when yielding starts.

$$\boxed{\frac{S_{yt}}{(fs)} = \sqrt{\left(\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2\right)}}$$



Region of Safety

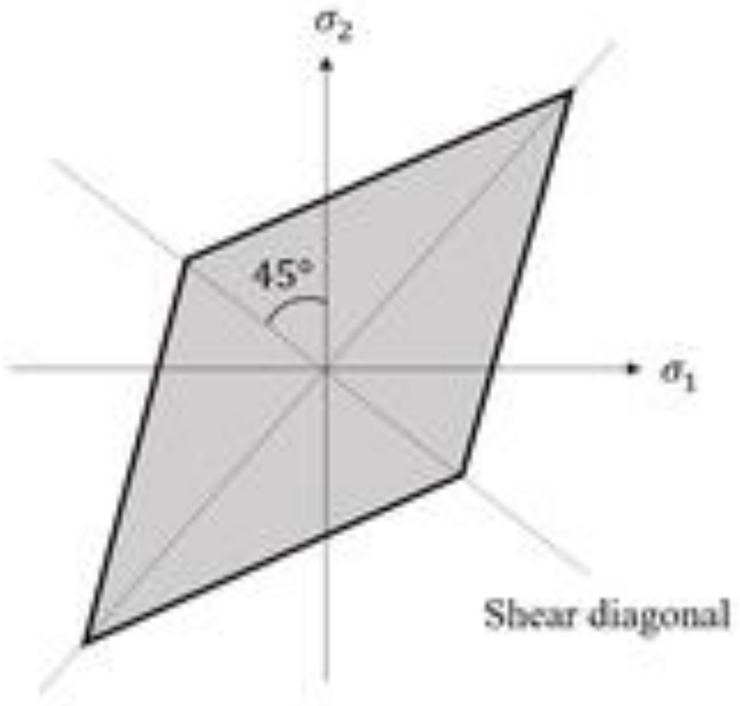
Maximum Principal Strain Theory

According to this theory, the failure or yielding occurs at a point in a member when the maximum principal (or normal) strain in a bi-axial stress system reaches the limiting value of strain (*i.e.* strain at yield point) as determined from a simple tensile test.

$$\varepsilon_1 = \varepsilon_{yt}$$

(or)

$$\boxed{\sigma_1 - \mu\sigma_2 = \frac{S_{yt}}{(fs)}}$$

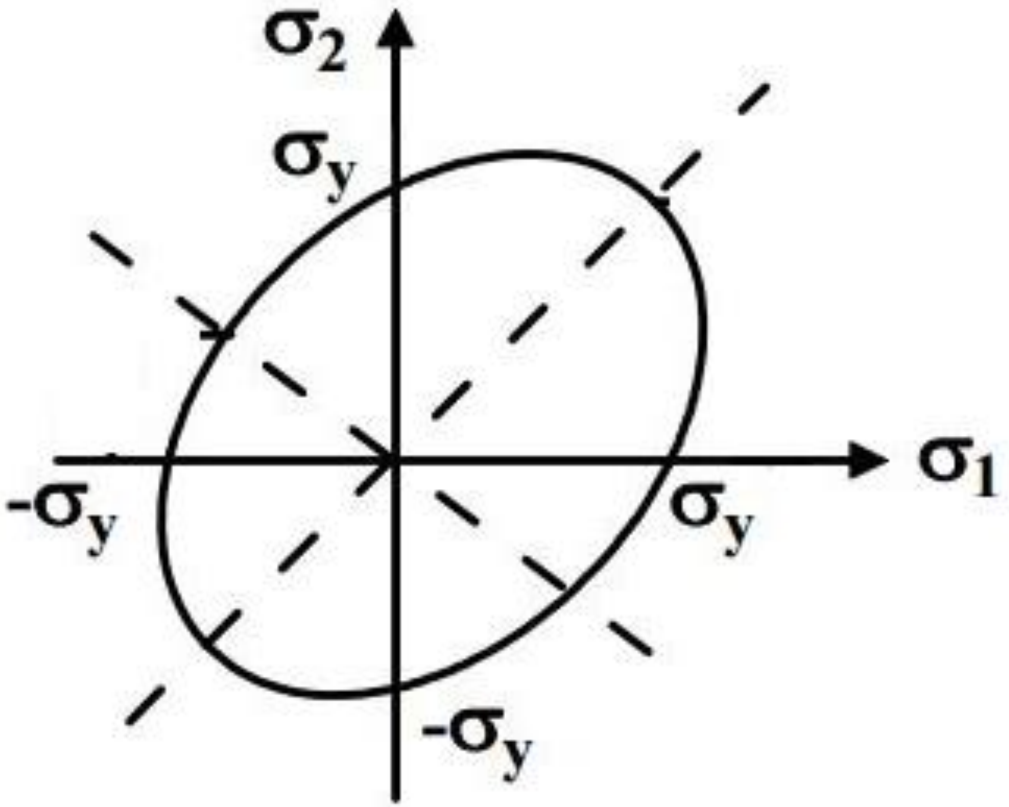


Region of Safety

Maximum Strain Energy Theory

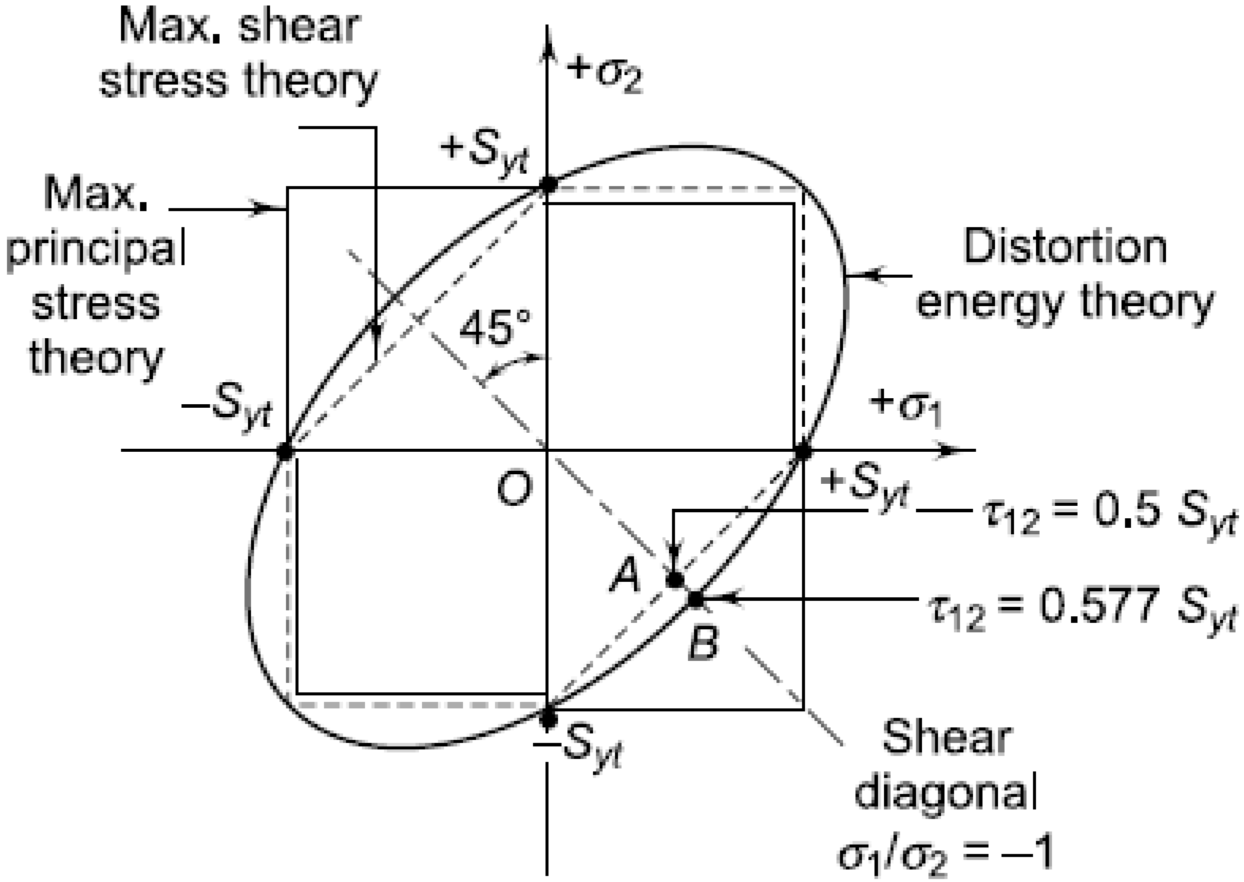
According to this theory, the failure or yielding occurs at a point in a member when the strain energy per unit volume in a bi-axial stress system reaches the limiting strain energy (*i.e.* strain energy at the yield point) per unit volume as determined from simple tension test.

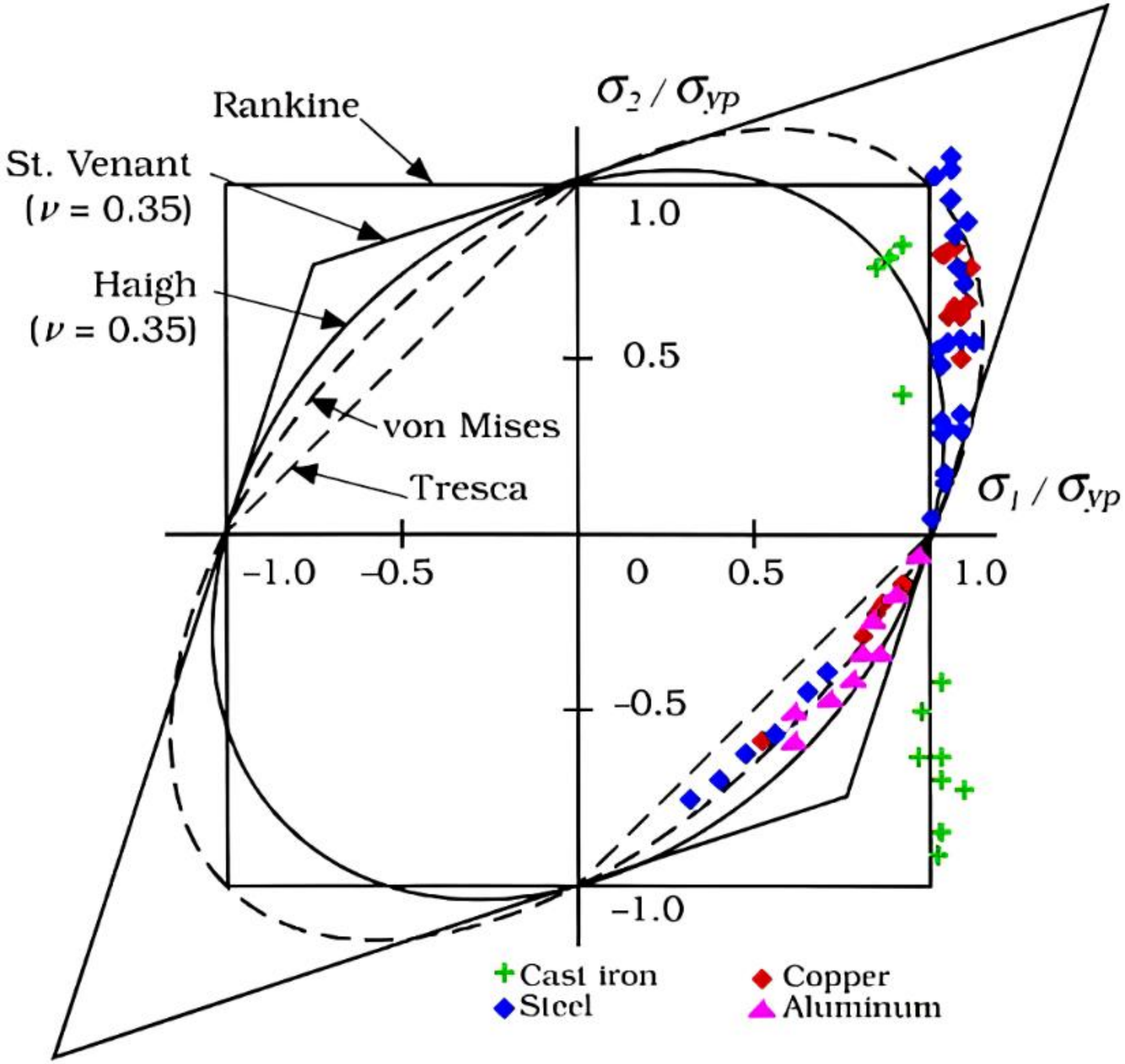
$$\boxed{\frac{S_{yt}}{(fs)} = \sqrt{(\sigma_1^2 - 2\mu\sigma_1\sigma_2 + \sigma_2^2)}}$$

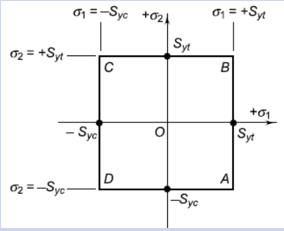
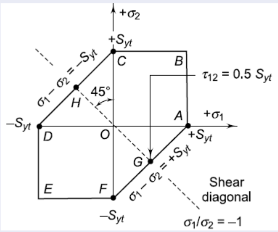
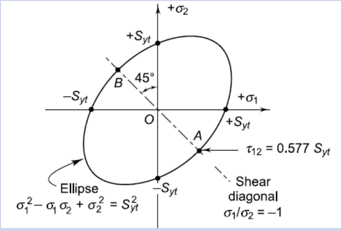
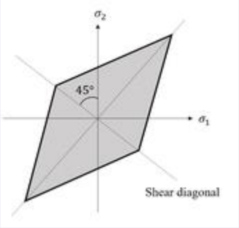
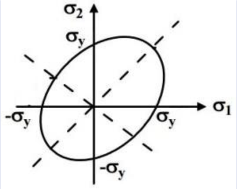


Region of Safety

Comparison of Theories of Elastic Failure





S.No.	Name of the Failure Theory	Governing Equation	Region of Safety	Shape of ROS
1	Maximum principal (or normal) stress theory (also known as Rankine's theory)	For tensile stresses, $\sigma_1 = \frac{S_{yt}}{(fs)}$ or $\sigma_1 = \frac{S_{ut}}{(fs)}$ For compressive stresses, $\sigma_1 = \frac{S_{yc}}{(fs)}$ or $\sigma_1 = \frac{S_{uc}}{(fs)}$		Square
2	Maximum shear stress theory (also known as Guest's or Tresca's theory)	$\sigma_1 - \sigma_2 = \frac{S_{yt}}{(fs)}$		Hexagon
3	Maximum distortion energy theory (also known as Hencky and Von Mises theory)	$\frac{S_{yt}}{(fs)} = \sqrt{(\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2)}$		Ellipse
4	Maximum principal (or normal) strain theory (also known as Saint Venant theory)	$\sigma_1 - \mu\sigma_2 = \frac{S_{yt}}{(fs)}$		Rhombus
5	Maximum strain energy theory (also known as Haigh's theory)	$\frac{S_{yt}}{(fs)} = \sqrt{(\sigma_1^2 - 2\mu\sigma_1\sigma_2 + \sigma_2^2)}$		Ellipse

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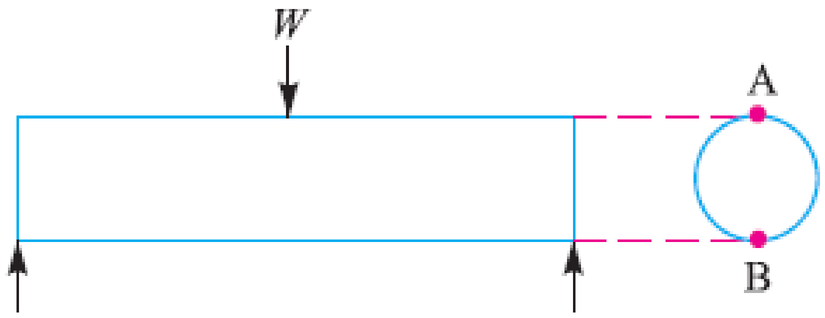
Design Against Fluctuating Loads

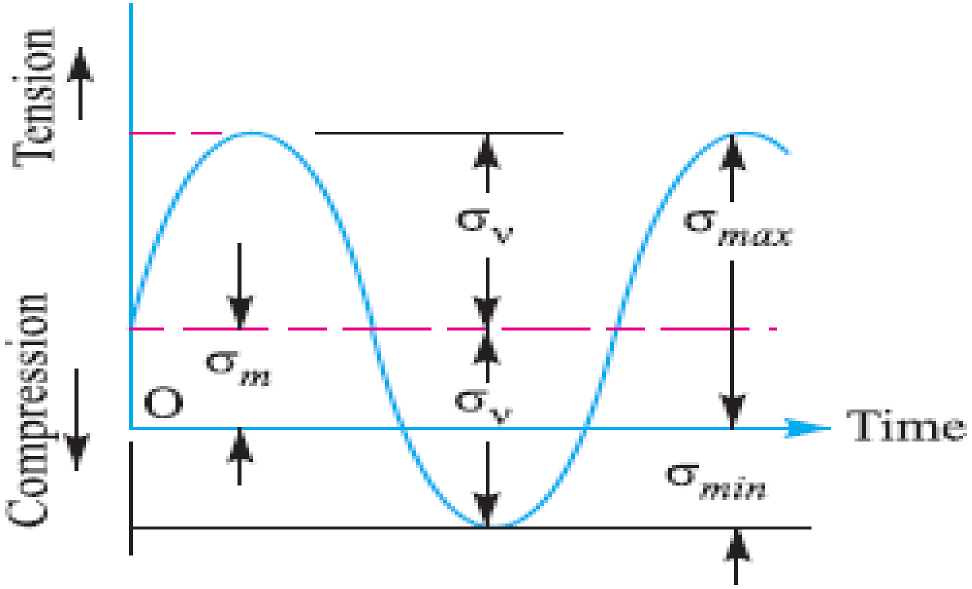
Fluctuating Stresses

- In many applications, the components are subjected to forces, which are not static, but vary in magnitude with respect to time. The stresses induced due to such forces are called *fluctuating stresses*.
- Consider a rotating beam of circular cross-section and carrying a load *W*, as shown in Figure aside. This load induces stresses in the beam which are cyclic in nature. *A* little consideration will show that the upper fibres of the beam (*i.e.* at point *A*) are under compressive stress and the lower fibres (*i.e.* at point *B*) are under tensile stress.

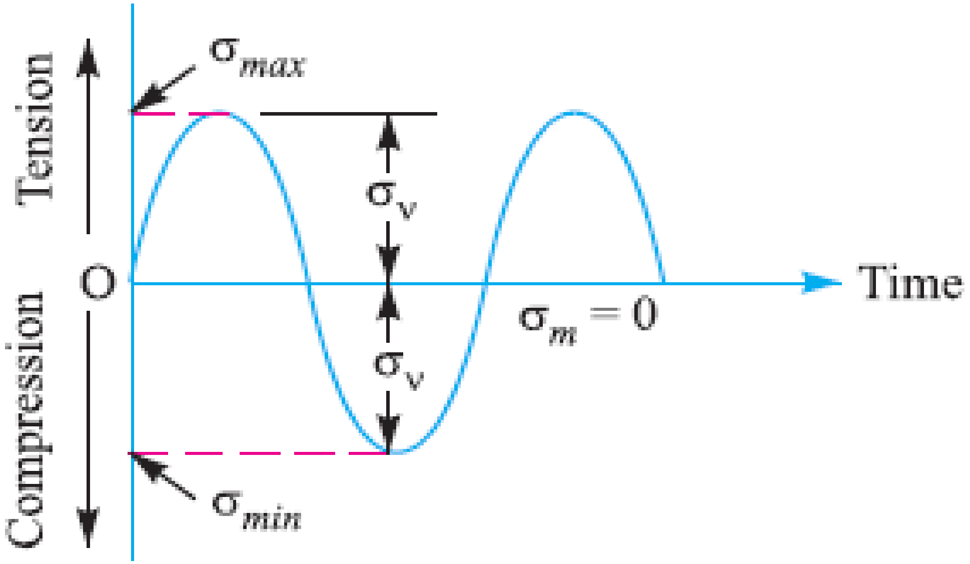


- After half a revolution, the point *B* occupies the position of point *A* and the point *A* occupies the position of point *B*. Thus the point *B* is now under compressive stress and the point *A* under tensile stress. The speed of variation of these stresses depends upon the speed of the beam.
- From above we see that for each revolution of the beam, the stresses are reversed from compressive to tensile. The stresses which vary from one value of compressive to the same value of tensile or *vice versa*, are known as *completely reversed or cyclic stresses.*

Different Types of Cyclic Stresses or Fluctuating Stresses



Fluctuating stress.



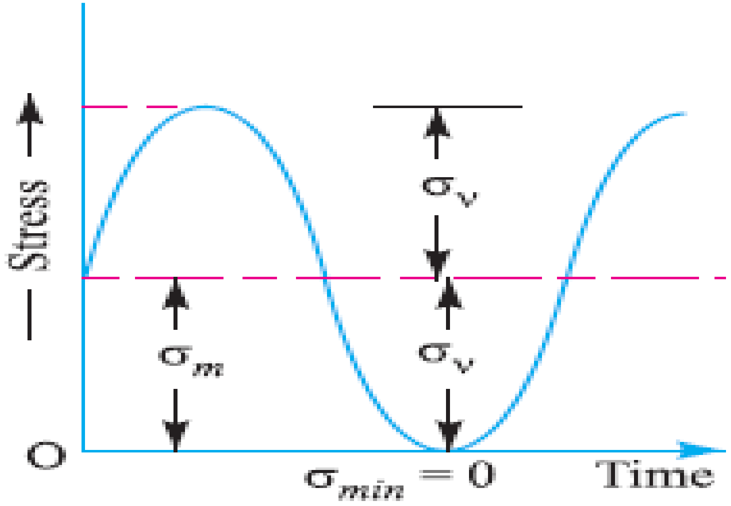
Completely reversed stress.

Mean or average stress,

$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2}$$

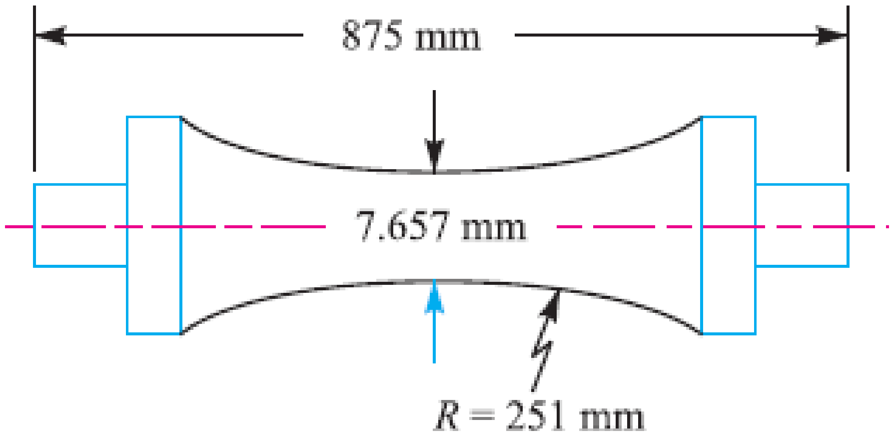
Reversed stress component or alternating or variable stress,

$$\sigma_v = \frac{\sigma_{max} - \sigma_{min}}{2}$$

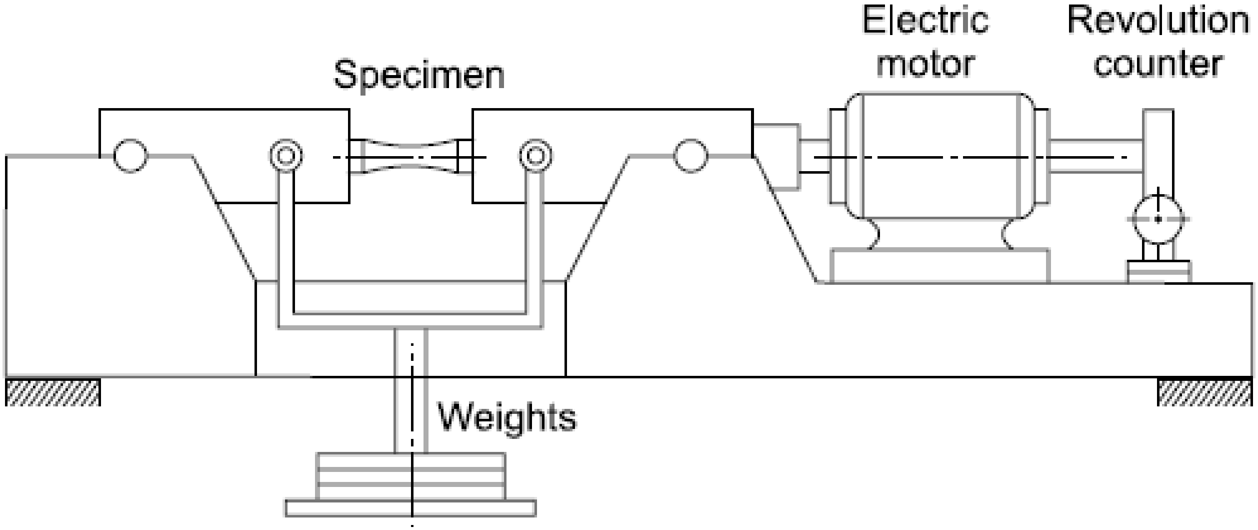


Repeated stress.

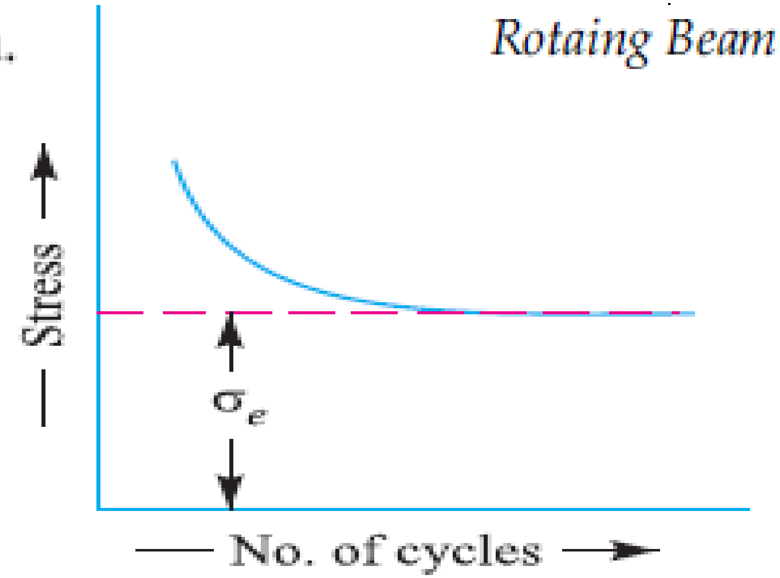
Fatigue and Endurance Limit



Standard specimen.



Rotaing Beam Fatigue Testing Machine



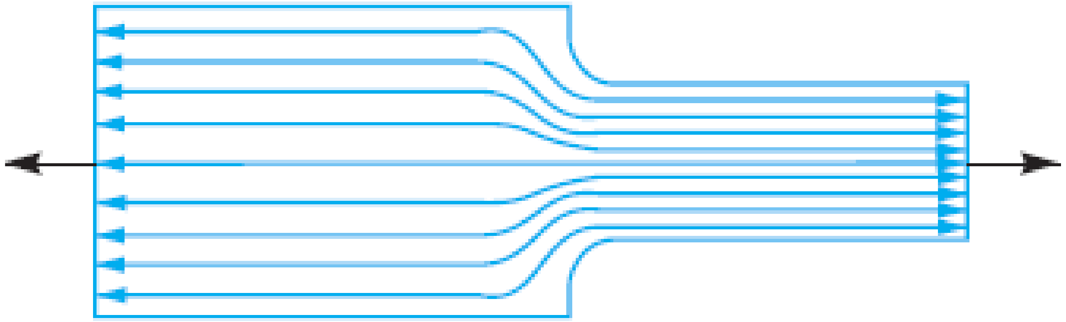
Endurance or fatigue limit.

Factor of Safety for Fatigue Loading

$$\text{Factor of safety } (F.S.) = \frac{\text{Endurance limit stress}}{\text{Design or working stress}} = \frac{\sigma_e}{\sigma_d}$$

Stress Concentration

Whenever a machine component changes the shape of its cross-section, the simple stress distribution no longer holds good and the neighbourhood of the discontinuity is different. This irregularity in the stress distribution caused by abrupt changes of form is called ***stress concentration*****.** It occurs for all kinds of stresses in the presence of fillets, notches, holes, keyways, splines, surface roughness or scratches etc.



Stress concentration.

Theoretical or Form Stress Concentration Factor

$$K_t = \frac{\text{Maximum stress}}{\text{Nominal stress}}$$

Fatigue Stress Concentration Factor

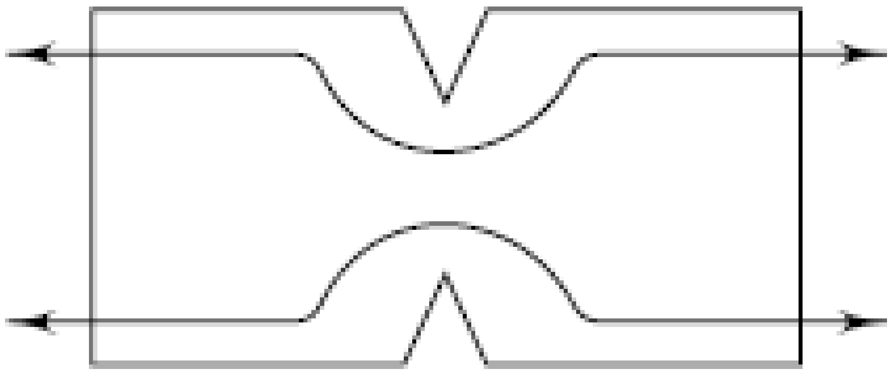
$$K_f = \frac{\text{Endurance limit without stress concentration}}{\text{Endurance limit with stress concentration}}$$

Notch Sensitivity

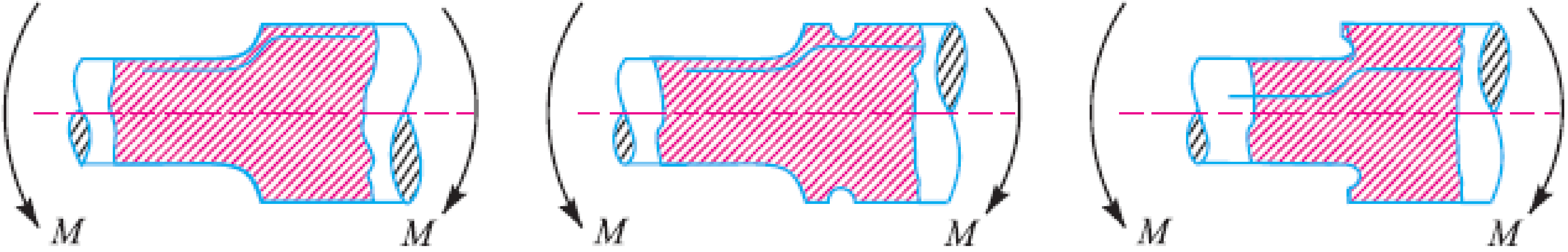
$$q = \frac{\text{Increase of actual stress over nominal stress}}{\text{Increase of theoretical stress over nominal stress}}$$

$$q = \frac{(K_f\sigma_o - \sigma_o)}{(K_t\sigma_o - \sigma_o)} \qquad q = \frac{(K_f - 1)}{(K_t - 1)} \qquad K_f = 1 + q(K_t - 1)$$

Methods of Reducing Stress Concentration



Additional notches and holes in the Tension Members

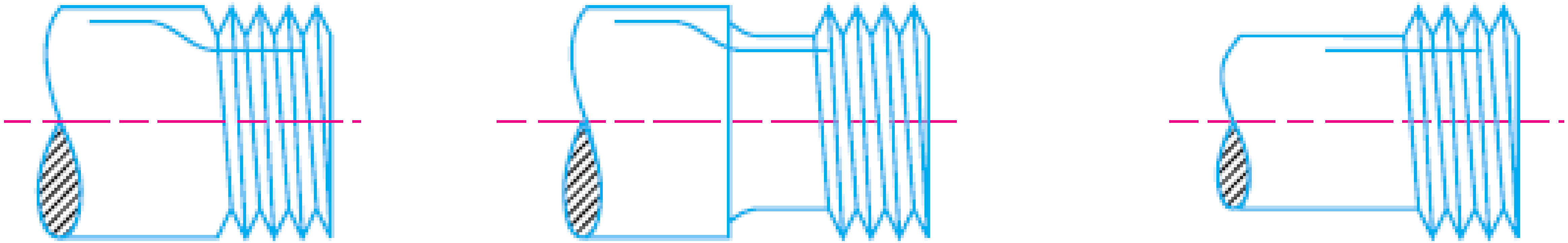


(*a*) Poor (*b*) Good (*c*) Preferred

Fillet Radius, under cutting and notch for members subjected to bending moment

(a) Poor (b) Preferred

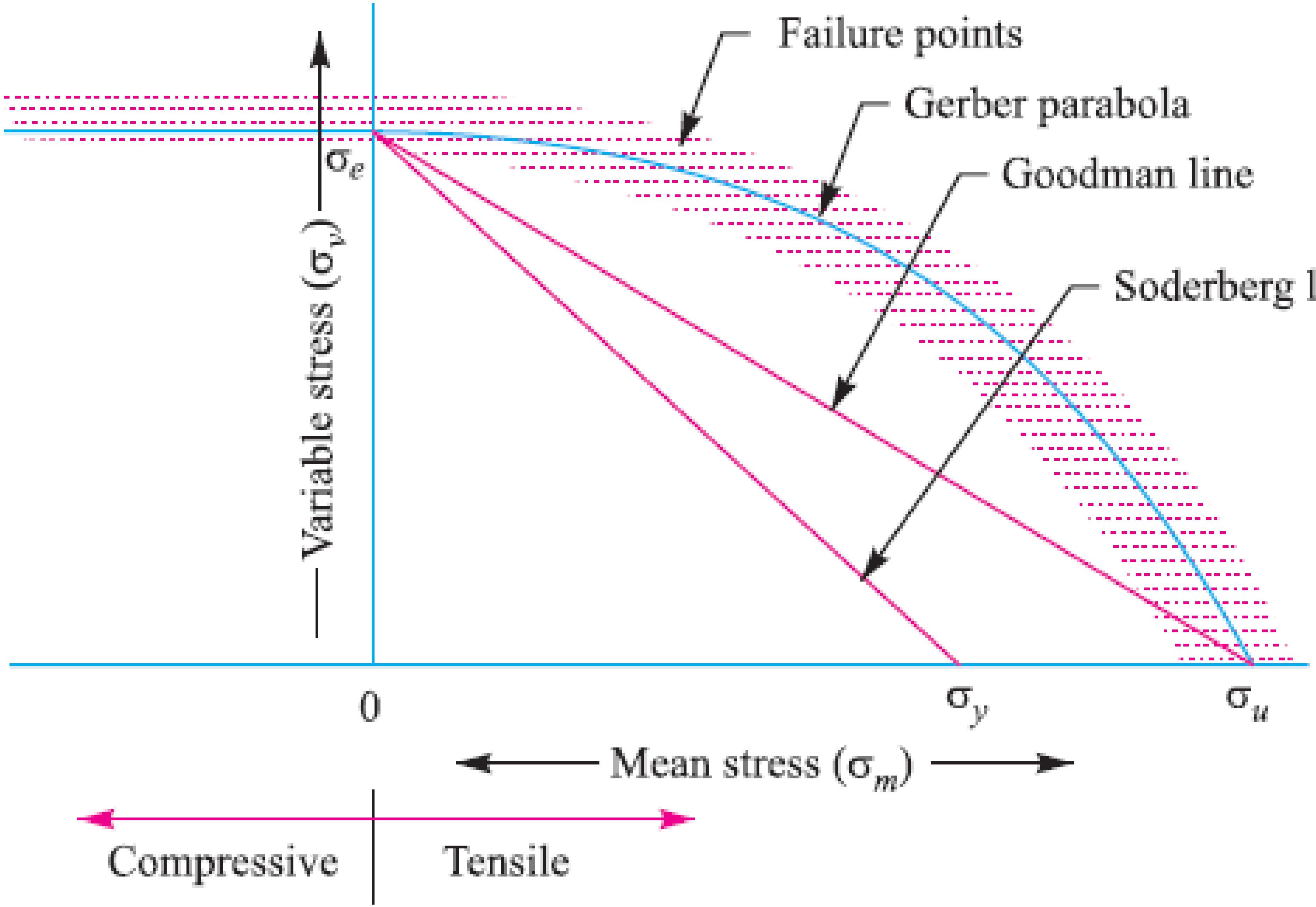
Drilling of Additional holes for shaft



(a) Poor (b) Good (c) Preferred

Reduction of stress concentration in threaded members

Design Against Combined Steady and Variable Stresses



Soderberg Method for Combination of Stresses

- The Soderberg method is particularly used for ductile materials.
- The following equation is applicable to ductile materials subjected to reversed bending load (tensile or compressive).

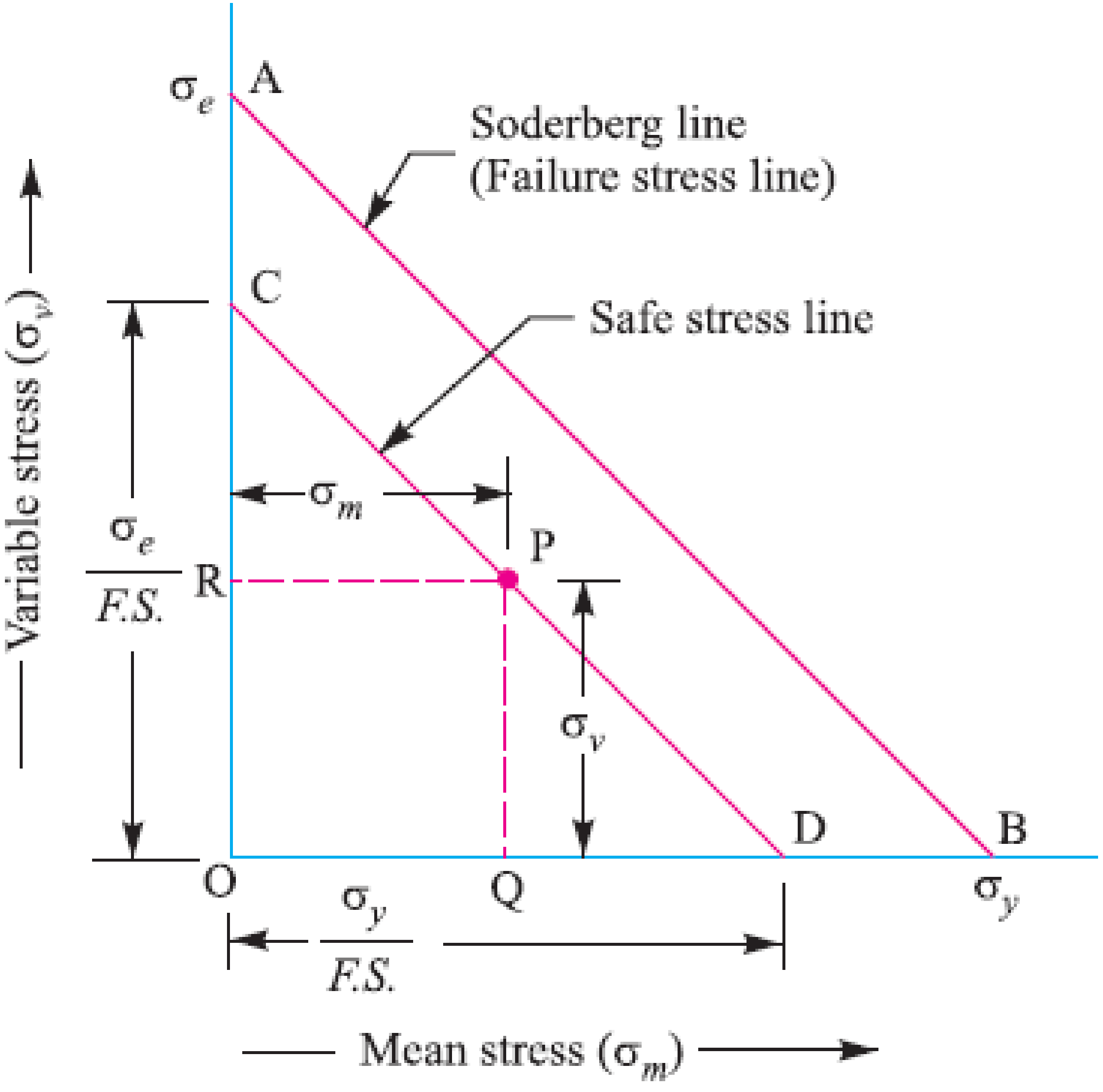
$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_y} + \frac{\sigma_v \times K_f}{\sigma_{eb} \times K_{sur} \times K_{sz}}$$

- When a machine component is subjected to reversed axial loading, then the above equation may be written as

$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_y} + \frac{\sigma_v \times K_f}{\sigma_{ea} \times K_{sur} \times K_{sz}}$$

- When a machine component is subjected to reversed shear loading, then equation may be written as

$$\frac{1}{F.S.} = \frac{\tau_m}{\tau_y} + \frac{\tau_v \times K_{fs}}{\tau_e \times K_{sur} \times K_{sz}}$$



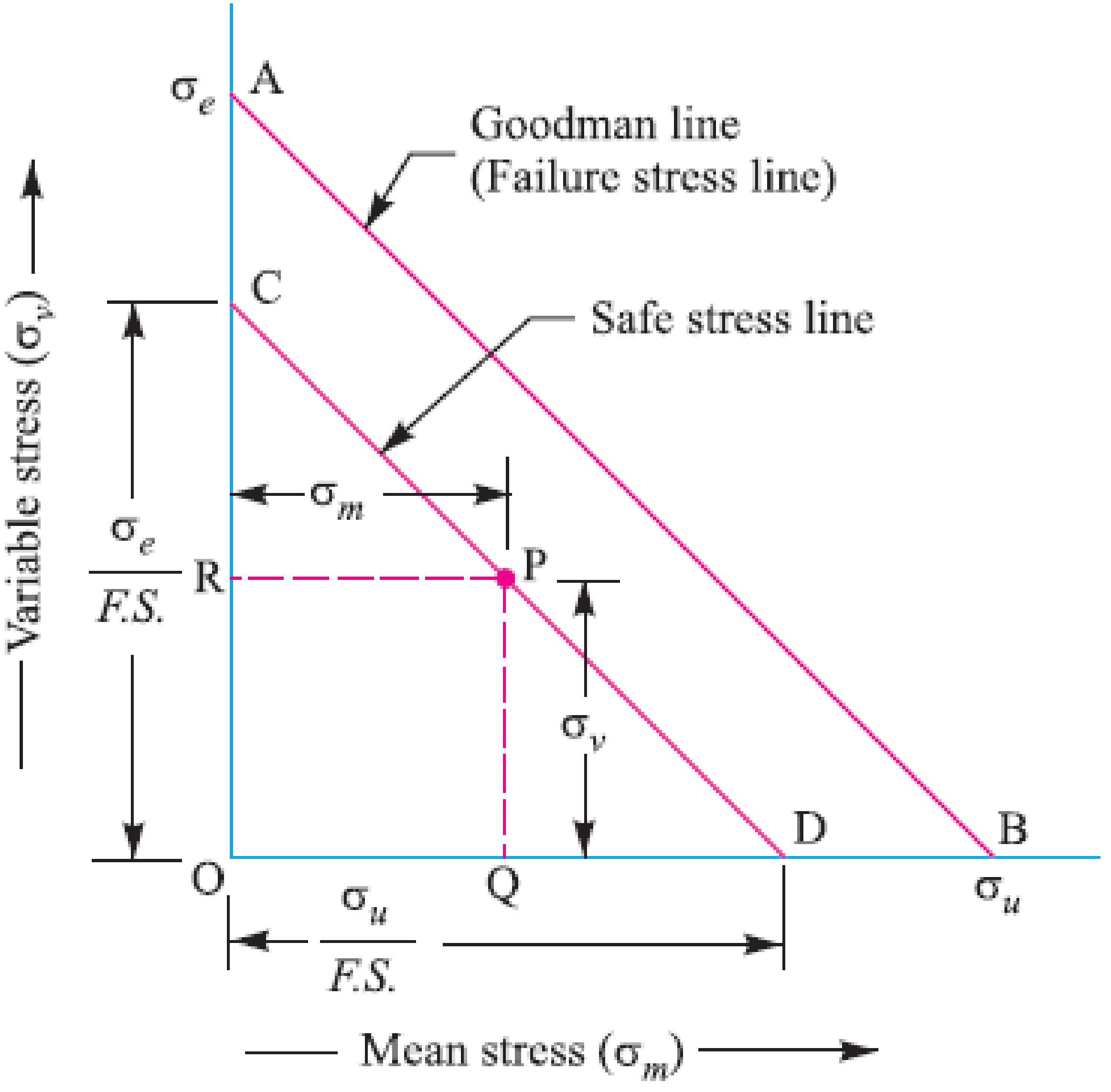
Goodman Method for Combination of Stresses

- A Goodman line is used when the design is based on ultimate strength and may be used for ductile or brittle materials.
- The following equation is applicable to ductile materials subjected to reversed bending loads (tensile or compressive).

$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_u} + \frac{\sigma_v \times K_f}{\sigma_{eb} \times K_{sur} \times K_{sz}} = \frac{\sigma_m}{\sigma_u} + \frac{\sigma_v \times K_f}{\sigma_e \times K_b \times K_{sur} \times K_{sz}}$$

- For brittle materials, the theoretical stress concentration factor (Kt) should be applied to the mean stress and fatigue stress concentration factor (Kf) to the variable stress. Thus for brittle materials, the equation

$$\frac{1}{F.S.} = \frac{\sigma_m \times K_t}{\sigma_u} + \frac{\sigma_v \times K_f}{\sigma_{eb} \times K_{sur} \times K_{sz}}$$



❖ **When a machine component is subjected to a load other than reversed bending, then the endurance limit for that type of loading should be taken into consideration. Thus for reversed axial loading (tensile or compressive), the following equations may be used for design**

$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_u} + \frac{\sigma_v \times K_f}{\sigma_{ea} \times K_{sur} \times K_{sz}} \quad \text{...(For ductile materials)}$$

$$\frac{1}{F.S.} = \frac{\sigma_m \times K_t}{\sigma_u} + \frac{\sigma_v \times K_f}{\sigma_{ea} \times K_{sur} \times K_{sz}} \quad \text{...(For brittle materials)}$$

❖ **Similarly, for reversed torsional or shear loading,**

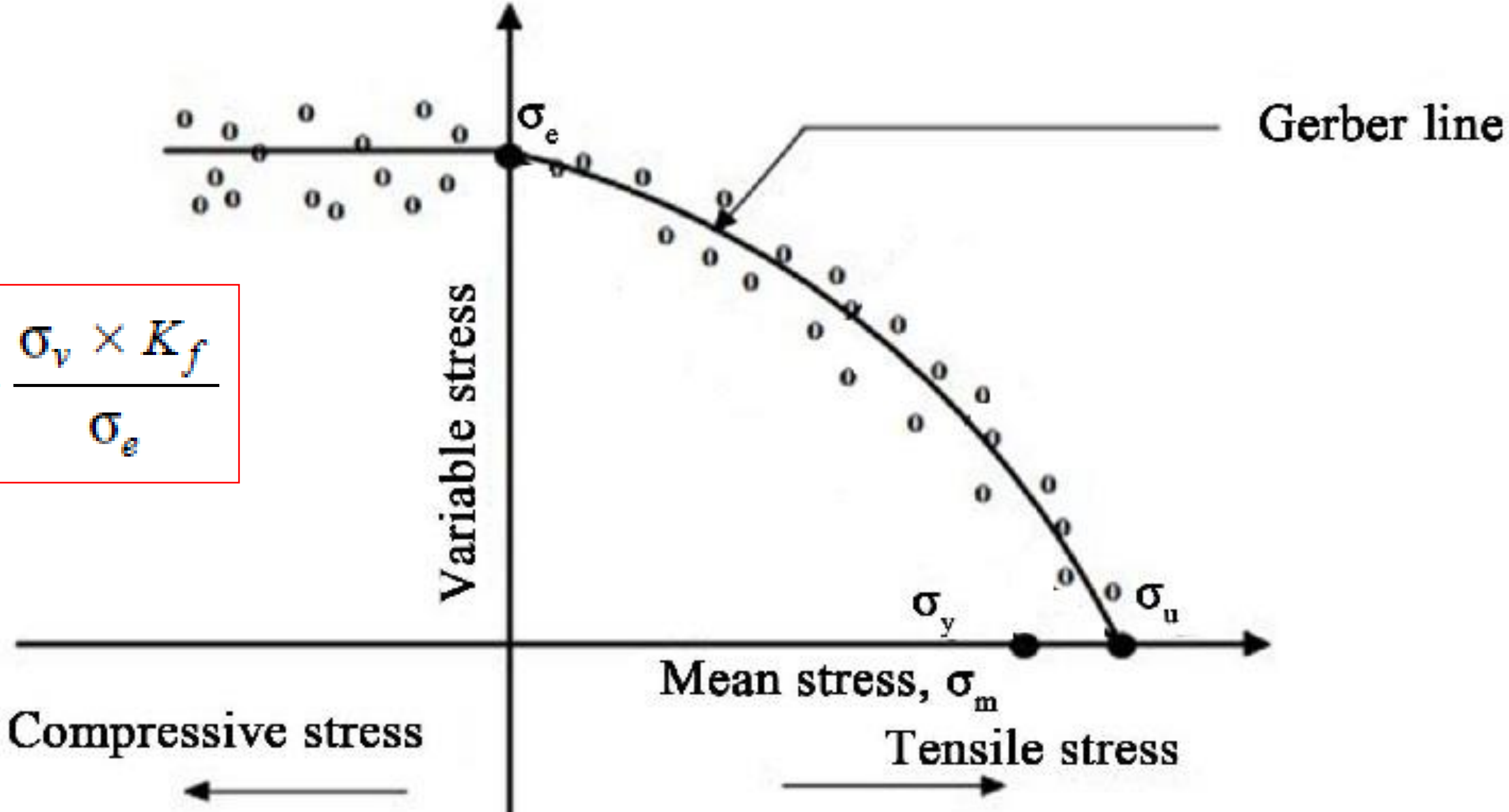
$$\frac{1}{F.S.} = \frac{\tau_m}{\tau_u} + \frac{\tau_v \times K_{fs}}{\tau_e \times K_{sur} \times K_{sz}} \quad \text{...(For ductile materials)}$$

$$\frac{1}{F.S.} = \frac{\tau_m \times K_{ts}}{\tau_u} + \frac{\tau_v \times K_{fs}}{\tau_e \times K_{sur} \times K_{sz}} \quad \text{...(For brittle materials)}$$

Gerber Method for Combination of Stresses

- **A parabolic curve drawn between the endurance limit (σe) and ultimate tensile strength (σu) was proposed by Gerber in 1874. Generally, the test data for ductile material fall closer to Gerber parabola as shown in Figure below, but because of scatter in the test points, a straight line relationship (i.e. Goodman line and Soderberg line) is usually preferred in designing machine parts.**

$$\boxed{\frac{1}{F.S.} = \left(\frac{\sigma_m}{\sigma_u}\right)^2 F.S. + \frac{\sigma_v \times K_f}{\sigma_e}}$$



Factors to be considered while designing machine parts to avoid fatigue failure

1. **The variation in the size of the component should be as gradual as possible.**
2. **The holes, notches and other stress raisers should be avoided.**
3. **The proper stress de-concentrators such as fillets and notches should be provided wherever necessary.**
4. **The parts should be protected from corrosive atmosphere.**
5. **A smooth finish of outer surface of the component increases the fatigue life.**
6. **The material with high fatigue strength should be selected.**
7. **The residual compressive stresses over the parts surface increases its fatigue strength.**

Combined Variable Normal Stress and Variable Shear Stress

When a machine part is subjected to both variable normal stress and a variable shear stress; then it is designed by using the following two theories of combined stresses :

1. Maximum shear stress theory, and 2. Maximum normal stress theory.

according to Soderberg's formula,

$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_y} + \frac{\sigma_v \times K_{fb}}{\sigma_{eb} \times K_{sur} \times K_{sz}} \quad \text{...(For reversed bending load)}$$

Multiplying throughout by σ_y, we get

$$\frac{\sigma_y}{F.S.} = \sigma_m + \frac{\sigma_v \times \sigma_y \times K_{fb}}{\sigma_{eb} \times K_{sur} \times K_{sz}}$$

The term on the right hand side of the above expression is known as ***equivalent normal stress*** due to reversed bending.

$\therefore$ Equivalent normal stress due to reversed bending,

$$\sigma_{neb} = \sigma_m + \frac{\sigma_v \times \sigma_y \times K_{fb}}{\sigma_{eb} \times K_{sur} \times K_{sz}} \quad \text{...}(i)$$

Similarly, equivalent normal stress due to reversed axial loading,

$$\sigma_{nea} = \sigma_m + \frac{\sigma_v \times \sigma_y \times K_{fa}}{\sigma_{ea} \times K_{sur} \times K_{sz}} \quad \text{...}(ii)$$

and total equivalent normal stress,

$$\sigma_{ne} = \sigma_{neb} + \sigma_{nea} = \frac{\sigma_y}{F.S.} \quad \text{...}(iii)$$

for reversed torsional or shear loading,

$$\frac{1}{F.S.} = \frac{\tau_m}{\tau_y} + \frac{\tau_v \times K_{fs}}{\tau_e \times K_{sur} \times K_{sz}}$$

Multiplying throughout by τ_y, we get

$$\frac{\tau_y}{F.S.} = \tau_m + \frac{\tau_v \times \tau_y \times K_{fs}}{\tau_e \times K_{sur} \times K_{sz}}$$

The term on the right hand side of the above expression is known as *equivalent shear stress.*

∴ Equivalent shear stress due to reversed torsional or shear loading,

$$\tau_{es} = \tau_m + \frac{\tau_v \times \tau_y \times K_{fs}}{\tau_e \times K_{sur} \times K_{sz}} \qquad \text{...}(iv)$$

The maximum shear stress theory is used in designing machine parts of ductile materials. According to this theory, maximum equivalent shear stress,

$$\tau_{es(max)} = \frac{1}{2}\sqrt{(\sigma_{ne})^2 + 4\,(\tau_{es})^2} = \frac{\tau_y}{F.S.}$$

The maximum normal stress theory is used in designing machine parts of brittle materials. According to this theory, maximum equivalent normal stress,

$$\sigma_{ne(max)} = \frac{1}{2}(\sigma_{ne}) + \frac{1}{2}\sqrt{(\sigma_{ne})^2 + 4\,(\tau_{es})^2} = \frac{\sigma_y}{F.S.}$$

Design of Joints

Cotter Joint

- **A cotter joint is used to *connect two co-axial rods*, which are subjected to either *axial tensile force* or *axial compressive force*.**
- **It is also used to connect a rod on one side with some machine part like a crosshead or base plate on the other side.**
- **It is not used for connecting shafts that rotate and transmit torque.**
- **Typical applications of cotter joint are as follows:**

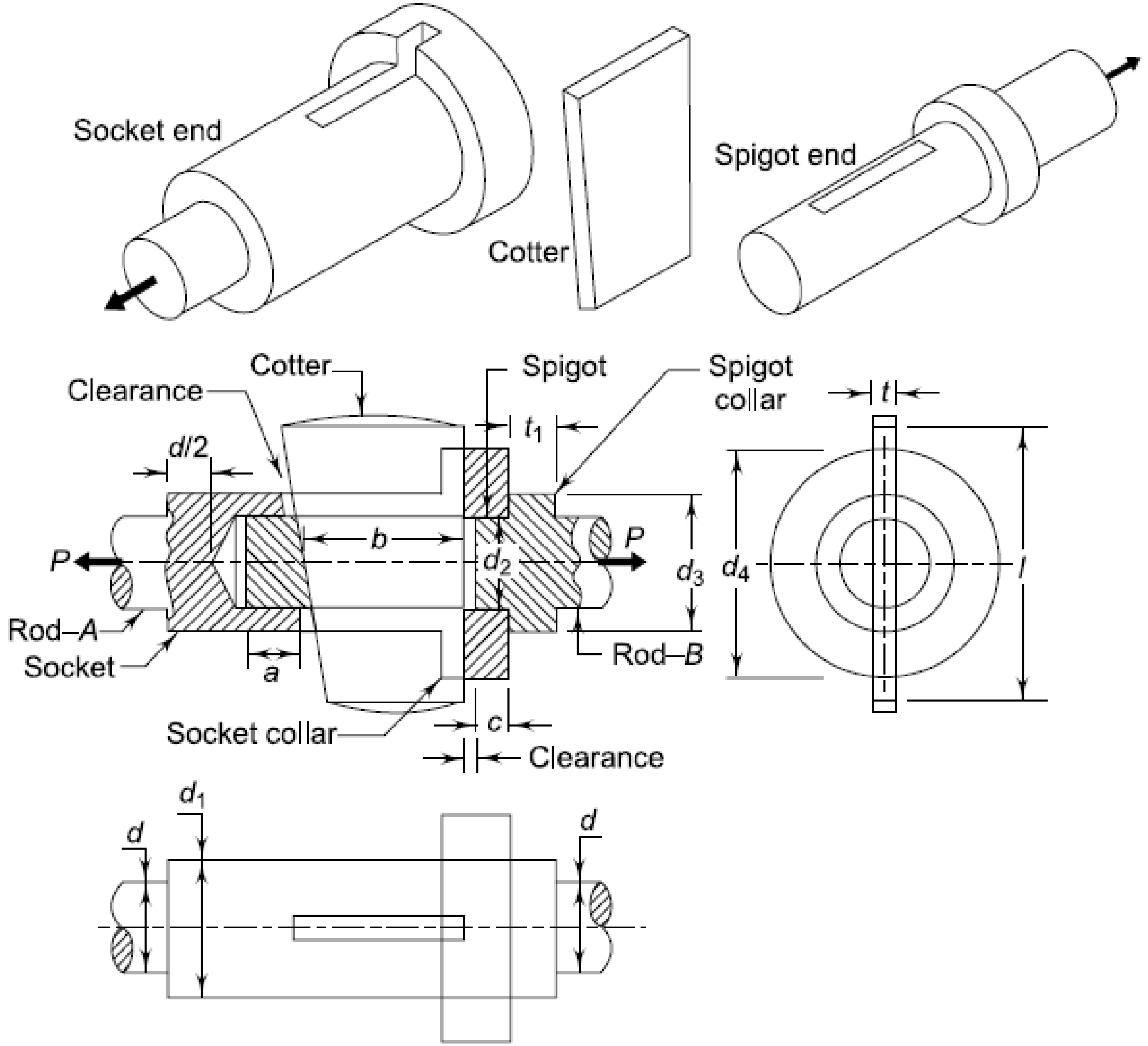
  (i) Joint between the piston rod and the crosshead of a steam engine

  (ii) Joint between the slide spindle and the fork of the valve mechanism

  (iii) Joint between the piston rod and the tail or pump rod

  (iv) Foundation bolt

P = tensile force acting on rods (N)
d = diameter of each rod (mm)
d_1 = outside diameter of socket (mm)
d_2 = diameter of spigot or inside diameter of socket (mm)
d_3 = diameter of spigot-collar (mm)
d_4 = diameter of socket-collar (mm)
a = distance from end of slot to the end of spigot on rod-B (mm)
b = mean width of cotter (mm)
c = axial distance from slot to end of socket collar (mm)
t = thickness of cotter (mm)
t_1 = thickness of spigot-collar (mm)
l = length of cotter (mm)



Design Procedure for Cotter Joint

(i) Calculate the diameter of each rod by the following Eq.

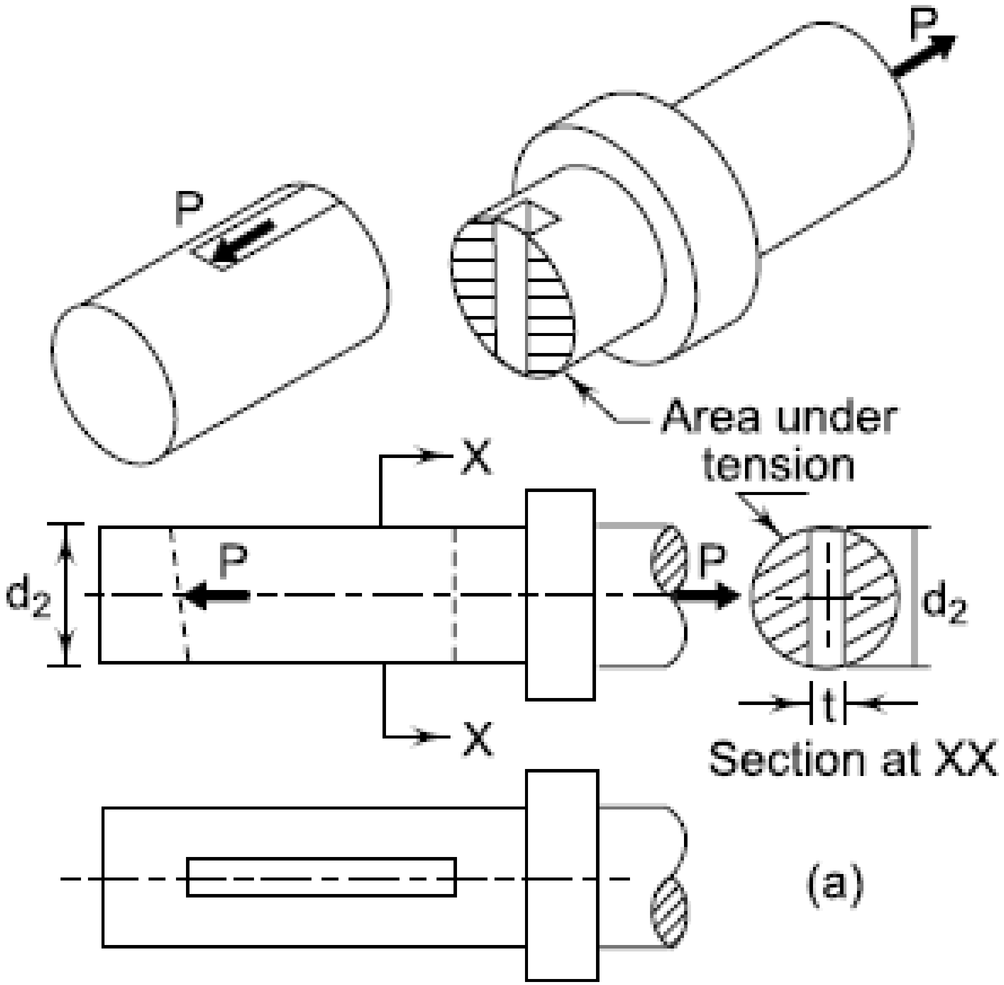
$$d = \sqrt{\frac{4P}{\pi \sigma_t}}$$

(ii) Calculate the thickness of the cotter by the empirical relationship given by

$$t = 0.31\, d$$

(iii) Calculate the diameter d_2 of the spigot on the basis of tensile stress using the following Eq.

$$P = \left[\frac{\pi}{4} d_2^2 - d_2\, t\right] \sigma_t$$



(iv) Calculate the outside diameter d_1 of the socket on the basis of tensile stress in the socket, from the Eq.

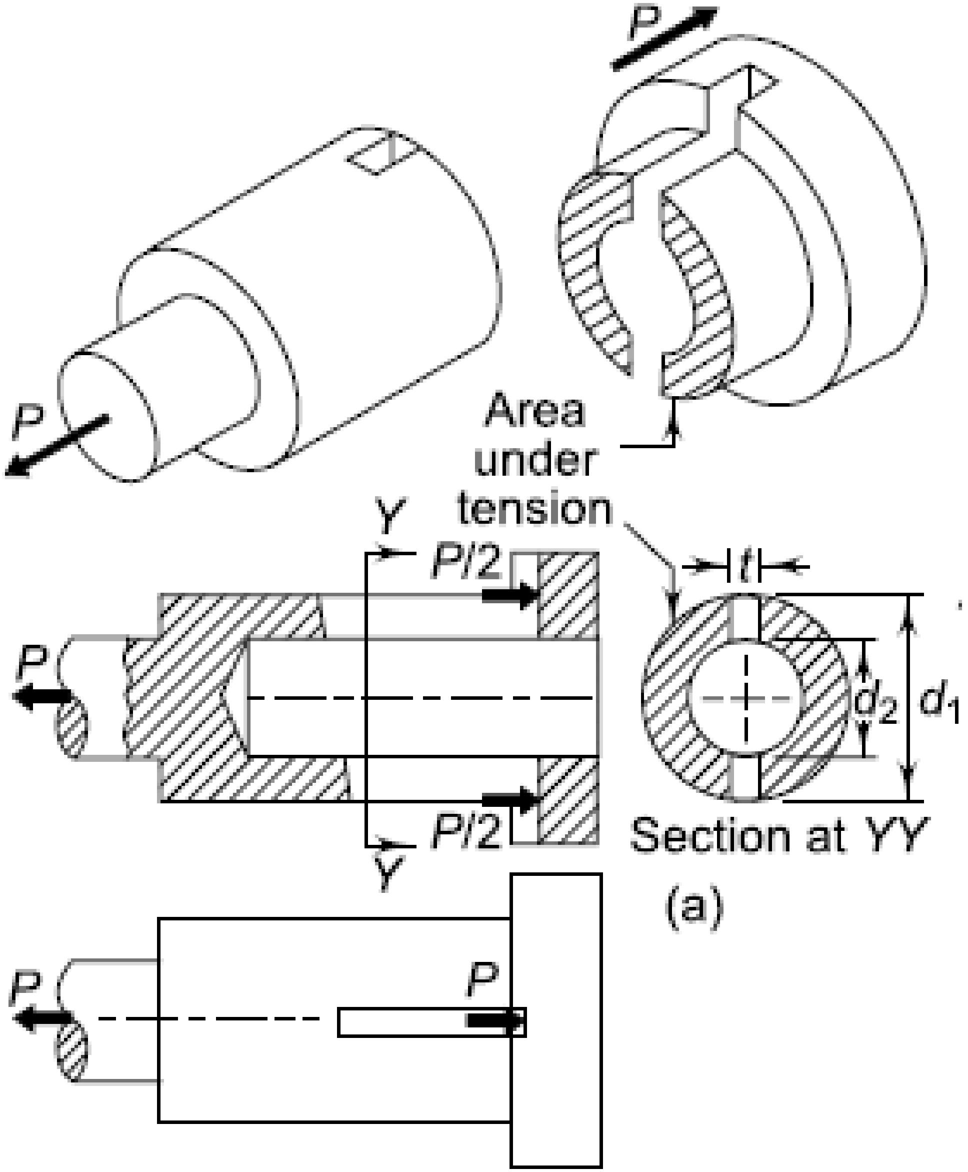
$$P = \left[\frac{\pi}{4}(d_1^2 - d_2^2) - (d_1 - d_2)t\right]\sigma_t$$

(v) The diameter of the spigot collar d_3 and the diameter of the socket collar d_4 are calculated by the following empirical relationships,

$$d_3 = 1.5\,d$$
$$d_4 = 2.4\,d$$

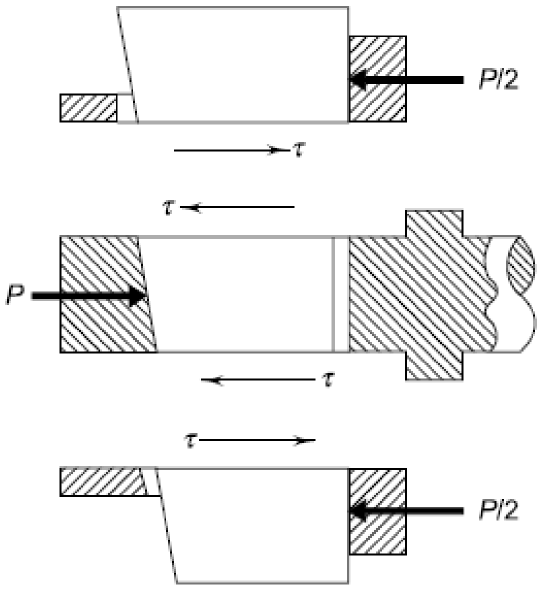
(vi) The dimensions a and c are calculated by the following empirical relationship,

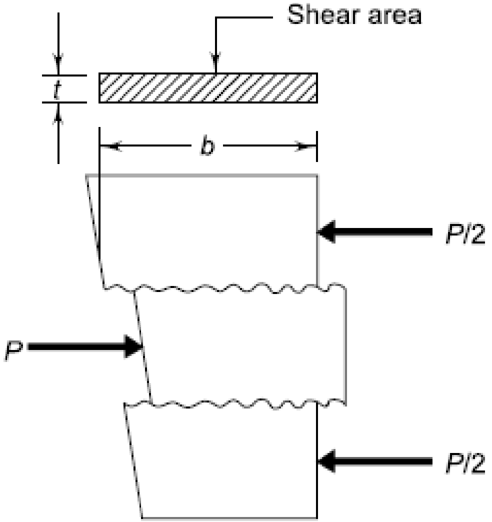
$$a = c = 0.75\,d$$

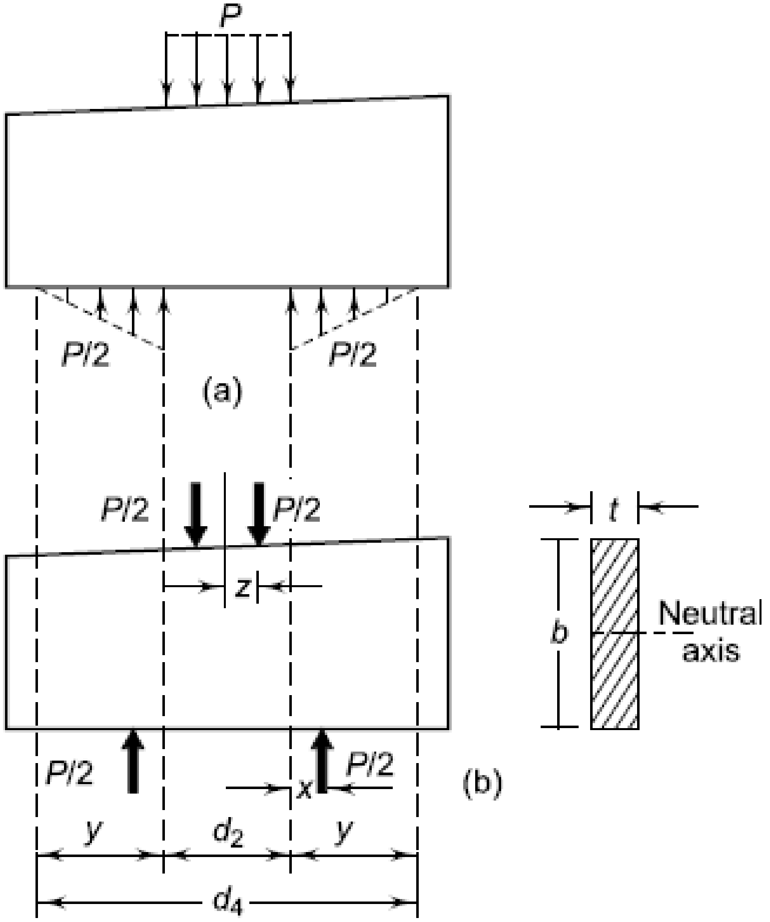


(vii) Calculate the width *b* of the cotter by shear consideration and bending consideration using the Eqs. and select the width, whichever is maximum between these two values.

$$b = \frac{P}{2\tau t} \qquad \text{or} \qquad b = \sqrt{\frac{3P}{t\sigma_b}\left[\frac{d_2}{4} + \frac{d_4 - d_2}{6}\right]}$$







$$M_b = \frac{P}{2}\left[\frac{d_2}{2} + x\right] - \frac{P}{2}(z)$$

$$= \frac{P}{2}\left[\frac{d_2}{2} + \frac{d_4 - d_2}{6}\right] - \frac{P}{2}\left[\frac{d_2}{4}\right]$$

$$= \frac{P}{2}\left[\frac{d_2}{4} + \frac{d_4 - d_2}{6}\right]$$

Also, $I = \dfrac{tb^3}{12}$ $\qquad y = \dfrac{b}{2}$

and $\sigma_b = \dfrac{M_b y}{I}$

Therefore,

$$\sigma_b = \frac{\frac{P}{2}\left[\frac{d_2}{4} + \frac{d_4 - d_2}{6}\right]\frac{b}{2}}{\left(\frac{tb^3}{12}\right)}$$

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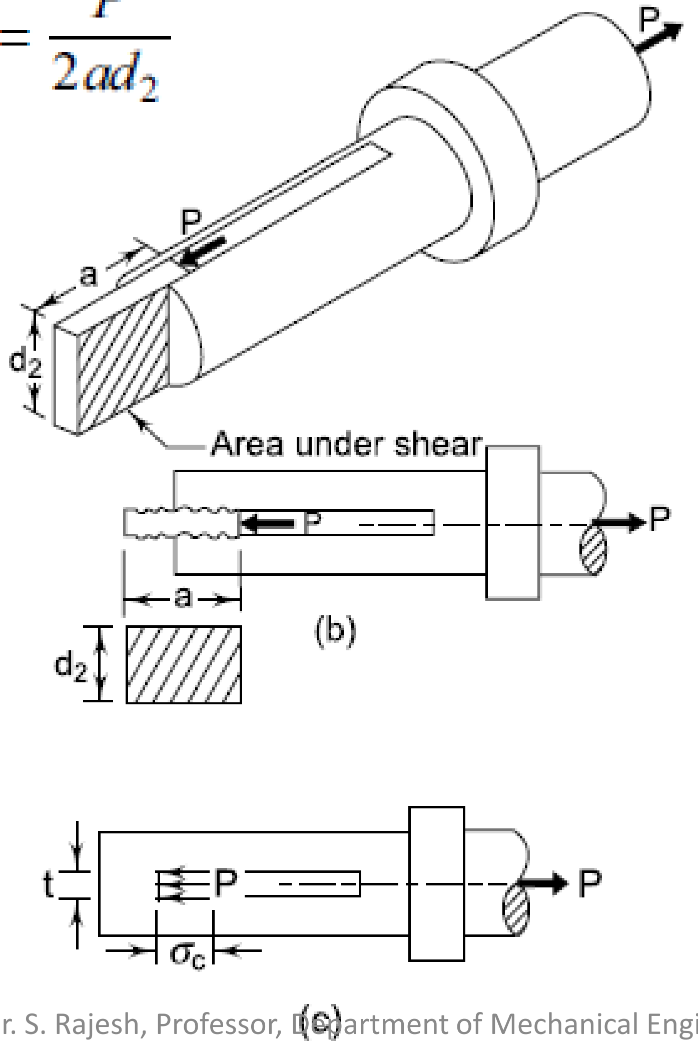
(viii) Check the crushing and shear stresses in the spigot end by the following Eqs.

$$\sigma_c = \frac{P}{td_2}$$

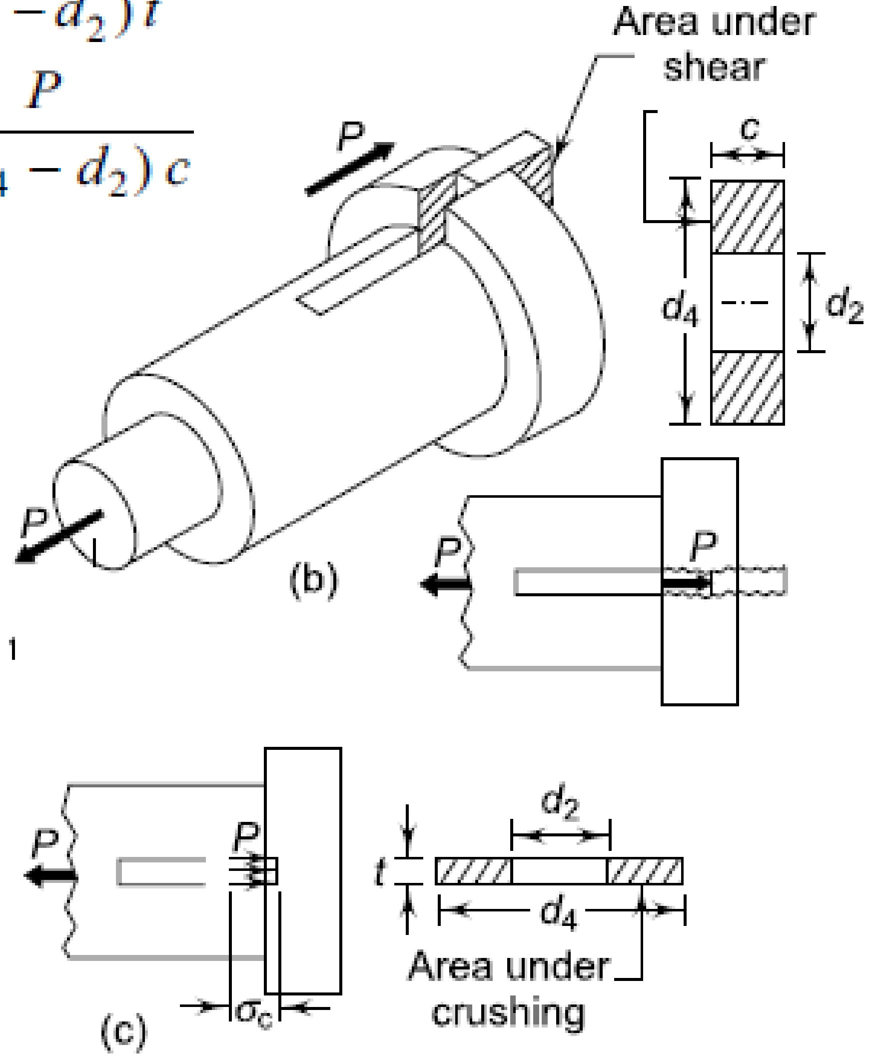
$$\tau = \frac{P}{2ad_2}$$



(ix) Check the crushing and shear stresses in the socket end by the following Eqs.

$$\sigma_c = \frac{P}{(d_4 - d_2)t}$$

$$\tau = \frac{P}{2(d_4 - d_2)c}$$

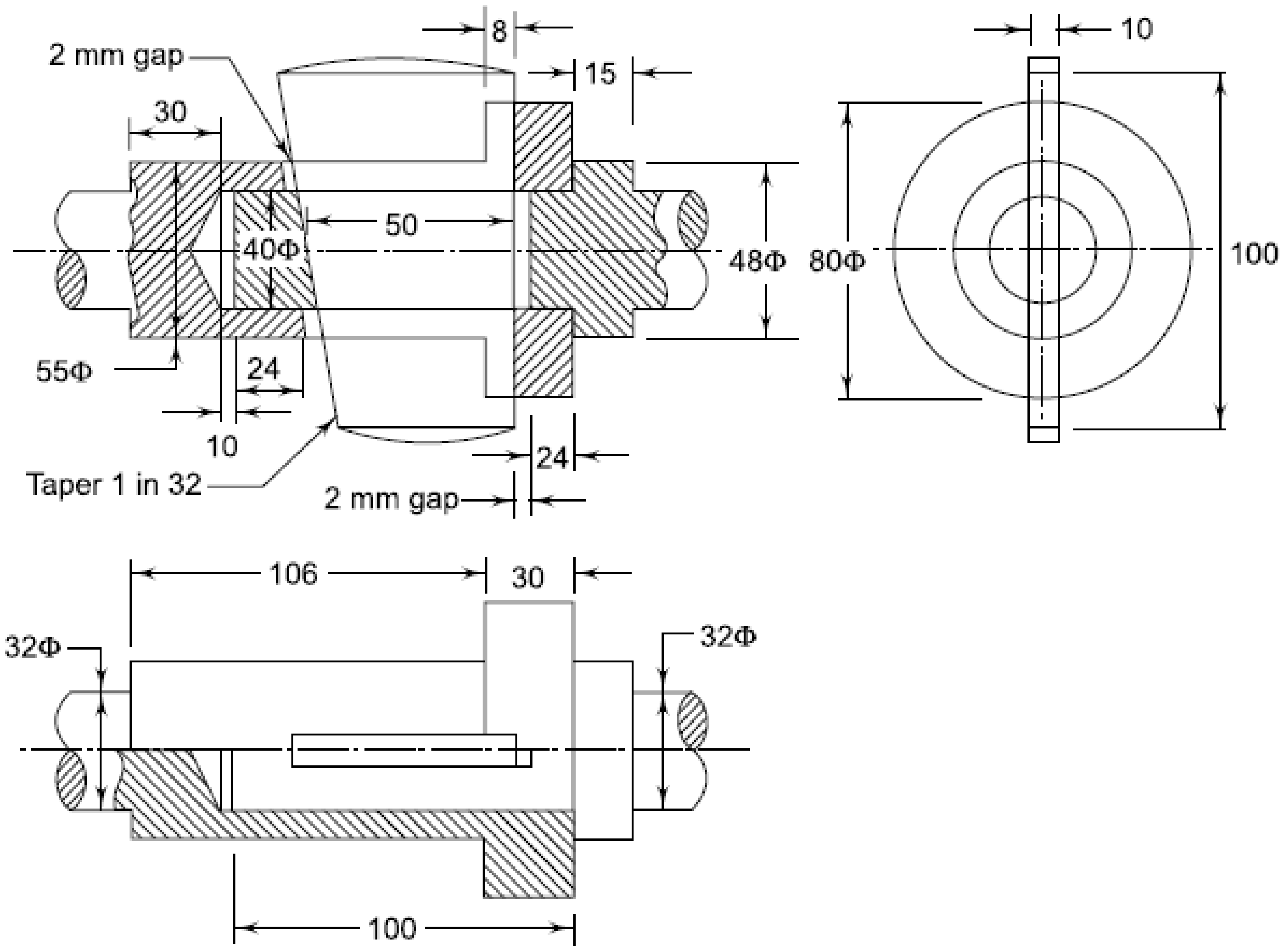


(x) Calculate the thickness *t*1 of the spigot collar by the following empirical relationship,

$$t_1 = 0.45\, d$$

The taper of the cotter is 1 in 32.

***Problem:* It is required to design a cotter joint to connect two steel rods of equal diameter. Each rod is subjected to an axial tensile force of 50 kN. Design the joint and specify its main dimensions. The material of the two rods and the cotter is selected as plain carbon steel of Grade 30C8 (S_{yt} = 400 N/mm^2). The factor of safety for the rods, spigot end and socket end is assumed as 6, while for the cotter, it is taken as 4. Assume S_{yc} = 2 S_{yt}.**



Knuckle Joint

- **Knuckle joint is used to *connect two rods* whose *axes either coincide* or *intersect* and lie in one plane. The knuckle joint is used to *transmit axial tensile force*.**
- **The construction of this *joint permits* limited *angular movement* between rods, about the axis of the pin.**
- **This type of joint is popular in machines and structures.**
- **Typical applications of knuckle joints are as follows:**

  (i) Joints between the tie bars in roof trusses.

  (ii) Joints between the links of a suspension bridge.

  (iii) Joints in valve mechanism of a reciprocating engine.

  (iv) Fulcrum for the levers.

  (v) Joints between the links of a bicycle chain.

D = diameter of each rod (mm)

D_1 = enlarged diameter of each rod (mm)

d = diameter of knuckle pin (mm)

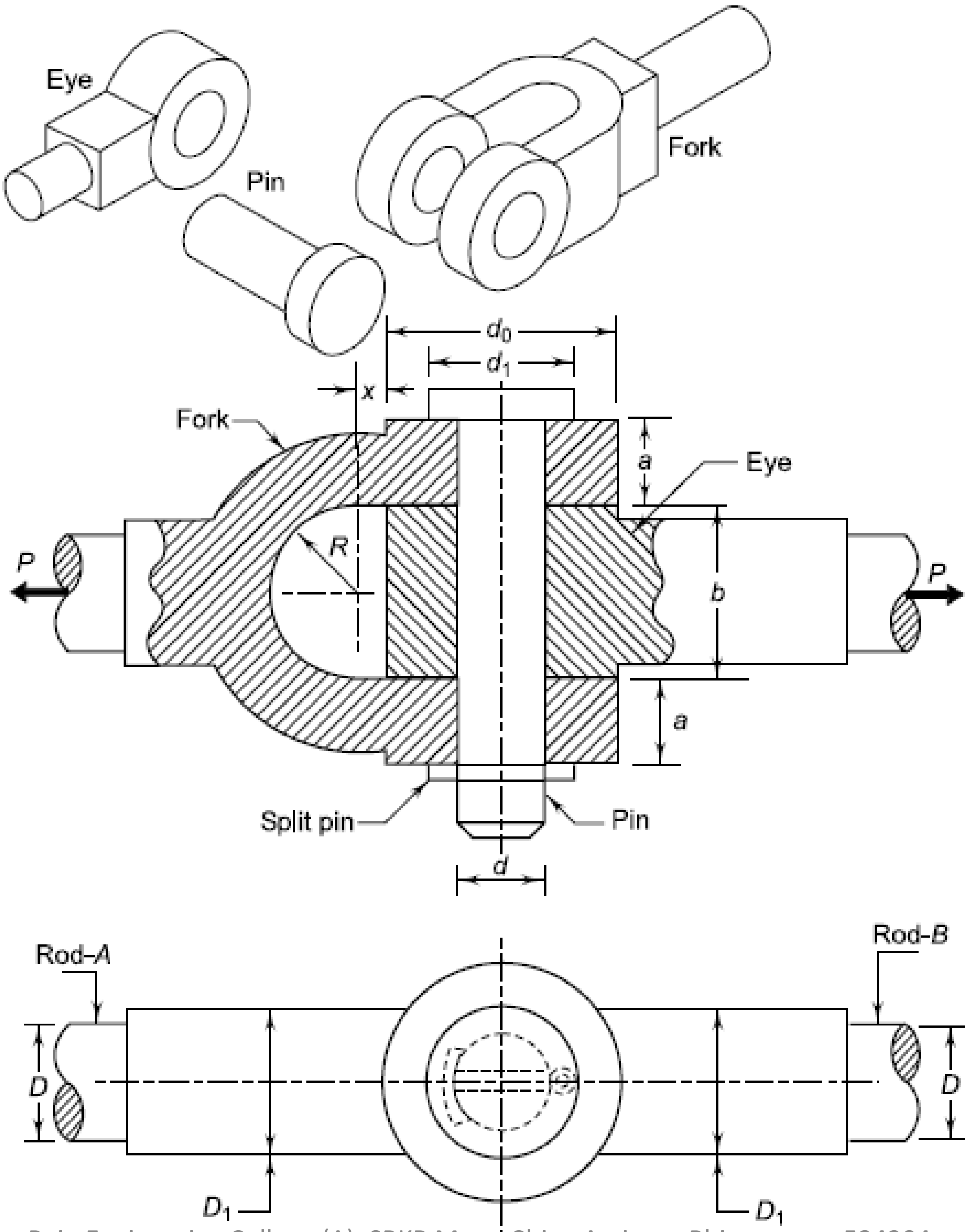
d_0 = outside diameter of eye or fork (mm)

a = thickness of each eye of fork (mm)

b = thickness of eye end of rod-B (mm)

d_1 = diameter of pin head (mm)

x = distance of the centre of fork radius R from the eye (mm)



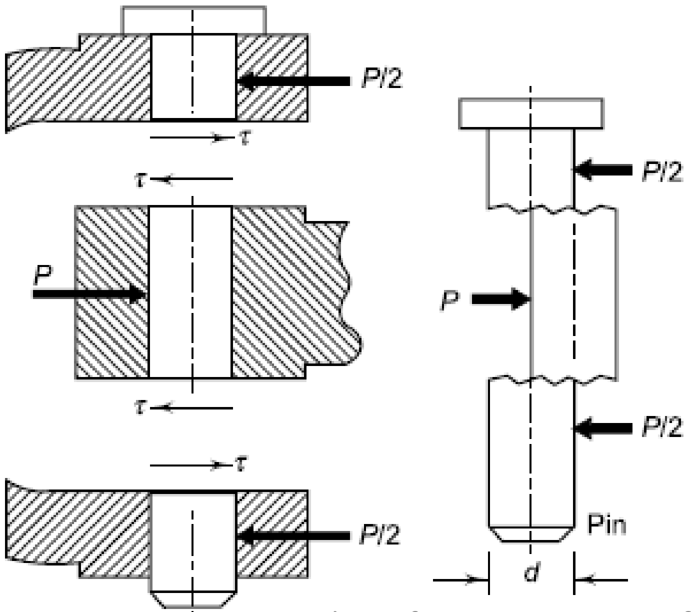
Design Procedure for Knuckle Joint

(i) Calculate the diameter of each rod by the following Equation $D = \sqrt{\dfrac{4P}{\pi\sigma_t}}$

(ii) Calculate the enlarged diameter of each rod by empirical relationship using Equation $D_1 = 1.1\,D$

(iii) Calculate the dimensions *a* and *b* by empirical relationship using the following Equations.
$a = 0.75\,D$ **&** $b = 1.25\,D$

(iv) Calculate the diameters of the pin by shear consideration and bending consideration and select the diameter, whichever is maximum.

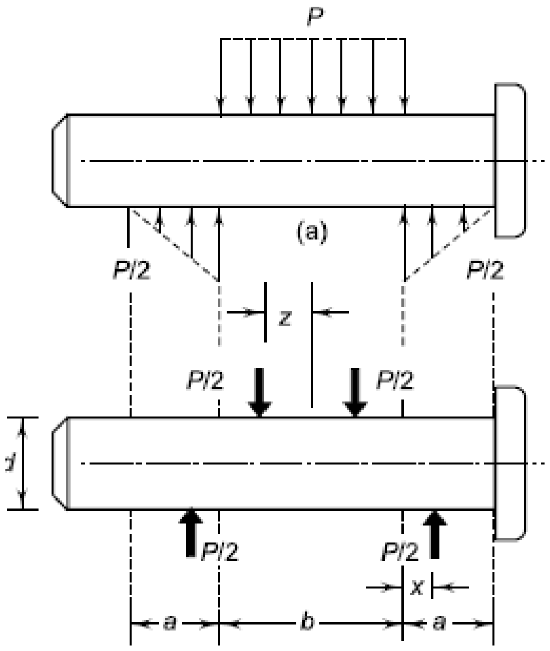


$$d = \sqrt{\frac{2P}{\pi\tau}}$$

or

$$d = \sqrt[3]{\frac{32}{\pi\sigma_b} \times \frac{P}{2}\left[\frac{b}{4} + \frac{a}{3}\right]}$$

(whichever is maximum)



$x = \frac{1}{3}a$ also, $z = \frac{1}{2}\left(\frac{1}{2}b\right) = \frac{1}{4}b$

The bending moment is maximum at the centre. It is given by,

$$M_b = \frac{P}{2}\left[\frac{b}{2} + x\right] - \frac{P}{2}(z)$$

$$= \frac{P}{2}\left[\frac{b}{2} + \frac{a}{3}\right] - \frac{P}{2}\left[\frac{b}{4}\right] = \frac{P}{2}\left[\frac{b}{4} + \frac{a}{3}\right]$$

Also, $I = \frac{\pi d^4}{64}$ and $y = \frac{d}{2}$

From Eq. (4.12),

$$\sigma_b = \frac{M_b y}{I} = \frac{\frac{P}{2}\left[\frac{b}{4} + \frac{a}{3}\right]\frac{d}{2}}{\frac{\pi d^4}{64}}$$

or $$\sigma_b = \frac{32}{\pi d^3} \times \frac{P}{2}\left[\frac{b}{4} + \frac{a}{3}\right]$$

(v) Calculate the dimensions d_o and d_1 by using following empirical relationships $d_o = 2d$ & $d_1 = 1.5d$

(vi) Check the tensile, crushing and shear stresses in the eye by using following equations

$$\sigma_t = \frac{P}{b\,(d_0 - d)}$$

$$\sigma_c = \frac{P}{bd}$$

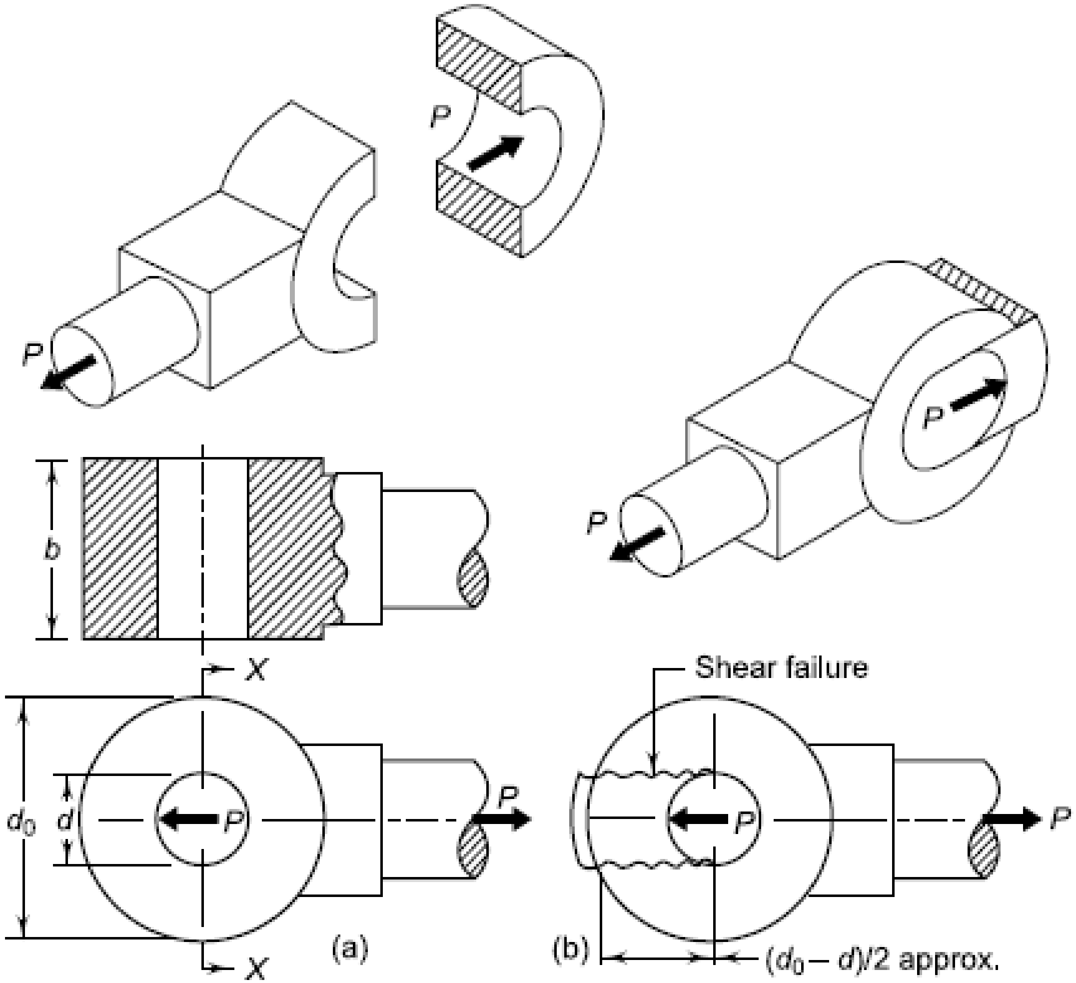
$$\tau = \frac{P}{b\,(d_0 - d)}$$

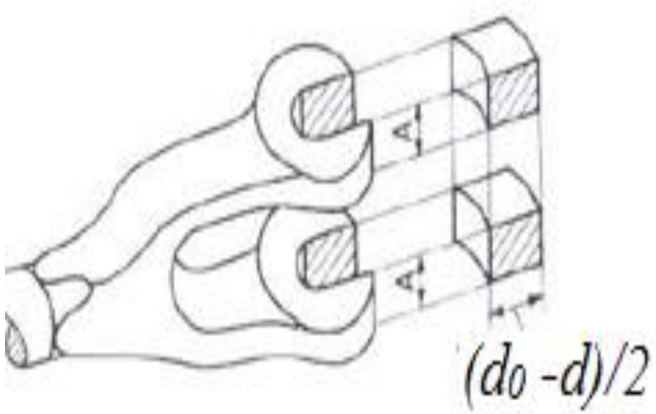
(vii) Check the tensile, crushing and shear stresses in the fork by using following equations

$$\sigma_t = \frac{P}{2a(d_0 - d)}$$

$$\sigma_c = \frac{P}{2ad}$$

$$\tau = \frac{P}{2a(d_0 - d)}$$





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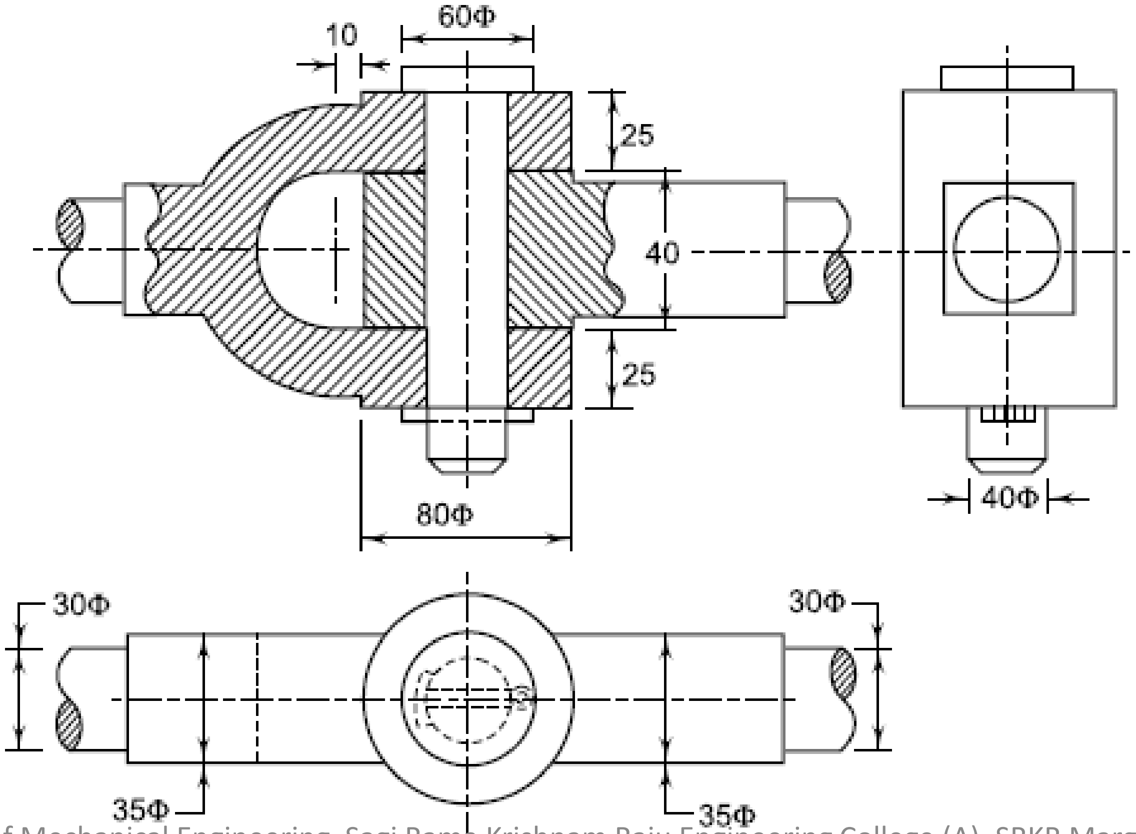
Problem: ***It is required to design a knuckle joint to connect two circular rods subjected to an axial tensile force of 50 kN. The rods are co-axial and a small amount of angular movement between their axes is permissible. Design the joint and specify the dimensions of its components. On strength basis, the material for two rods and pin may be selected as plain carbon steel of Grade 30C8 (S_{yt} = 400 N/mm^2). It is further assumed that the yield strength in compression is equal to yield strength in tension. In practice, the compressive strength of steel is much higher than its tensile strength. The factor of safety for all parts may be assumed as 5.***



Design of Threaded Joints

Threaded Joints

❖ ***Threaded joint is defined as a separable joint of two or more machine parts that are held together by means of a threaded fastening such as a bolt and a nut.***

❖ **The salient features of this definition are as follows:**

i. **Threaded joints are used to hold two or more machine parts together. These parts can be dismantled, if required, without any damage to machine parts or fastening. Therefore, threaded joints are *detachable joints*, *unlike* welded joints.**

ii. **Thread is the basic element of these joints. The thread is formed by cutting a helical groove on the surface of a cylindrical rod or cylindrical hole. The threaded element can take the shape of bolt and nut, screw or stud. Sometimes, threads are cut on the parts to be joined.**

Advantages of Threaded Joints

The advantages of threaded joints are as follows:

i. **The parts are held together by means of a large clamping force. There is wedge action at the threads, which increases the clamping force. There is no loosening of the parts. Therefore, threaded joints are '*reliable*' joints.**

ii. **The parts are assembled by means of a spanner. The length of the spanner is large compared with the radius of the thread. Therefore, the mechanical advantage is more and force required to tighten the joint is small.**

iii. **Threaded joints have small overall dimensions resulting in compact construction.**

iv. **The threads are self-locking. Therefore, threaded joints can be placed in any position—vertical, horizontal or inclined.**

v. **Threaded fasteners are economical to manufacture. Their manufacturing is simple. High accuracy can be maintained for the threaded components.**

vi. **The parts joined together by threaded joints can be detached as and when required. This requirement is essential in certain applications for the purpose of inspection, repair or replacement.**

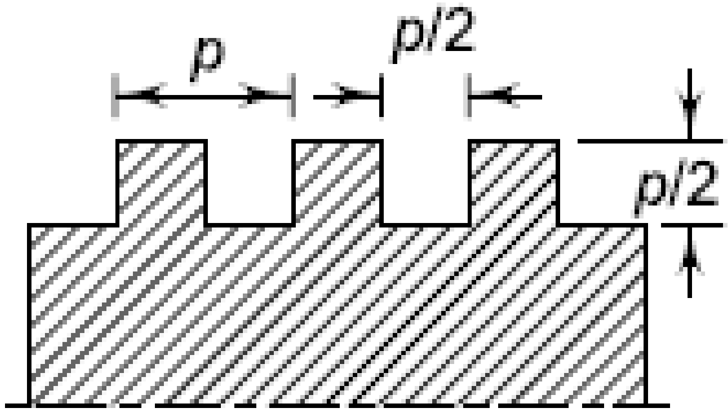
vii. **Threaded fasteners are standardised and a wide variety is available for different operating conditions and applications.**

Disadvantages of Threaded Joints

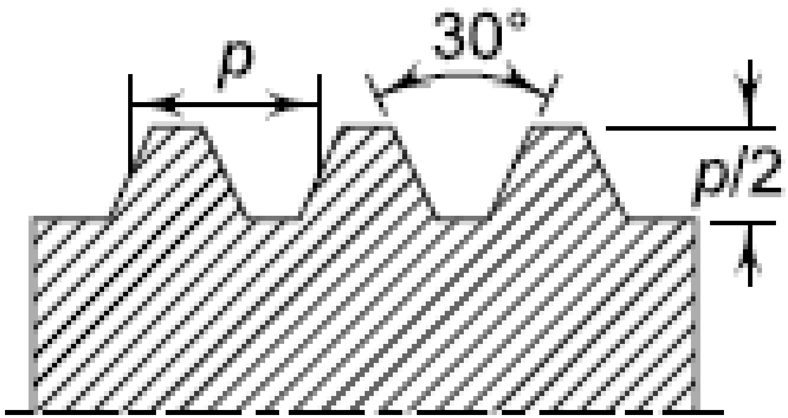
There are certain disadvantages of threaded joints. They are as follows:

i. **Threaded joints require holes in the machine parts that are to be clamped. This results in stress concentration near the threaded portion of the parts. Such areas are vulnerable to fatigue failure.**
ii. **Threaded joints loosen when subjected to vibrations.**
iii. **Threaded fasteners are considered as a major obstacle for efficient assembly.** ***In manual assembly, the cost of tightening a screw can be six to ten times the cost of the screw itself. Therefore, Design for Manufacture and Assembly (DFMA) recommends*** **minimum number of threaded fasteners.**

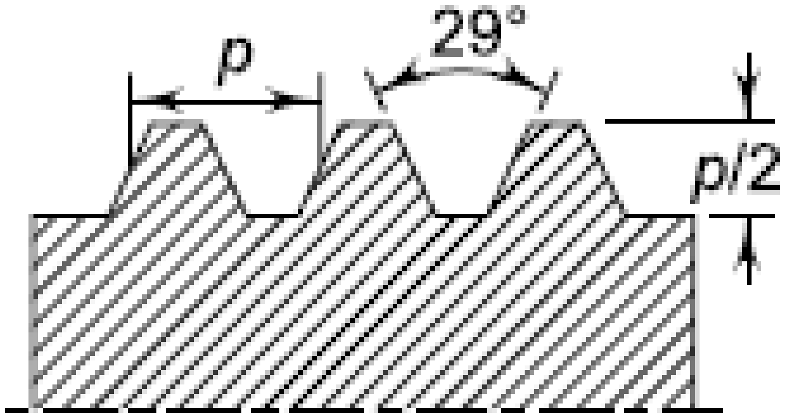
Forms of Threads



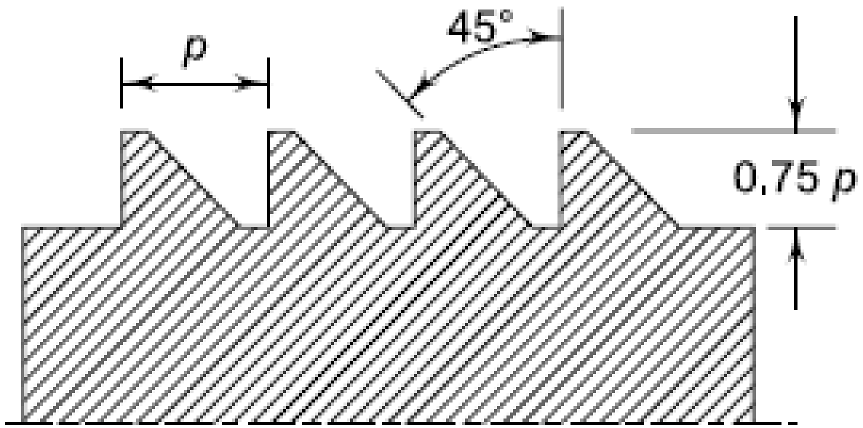
Square Threads



Trapezoidal Threads

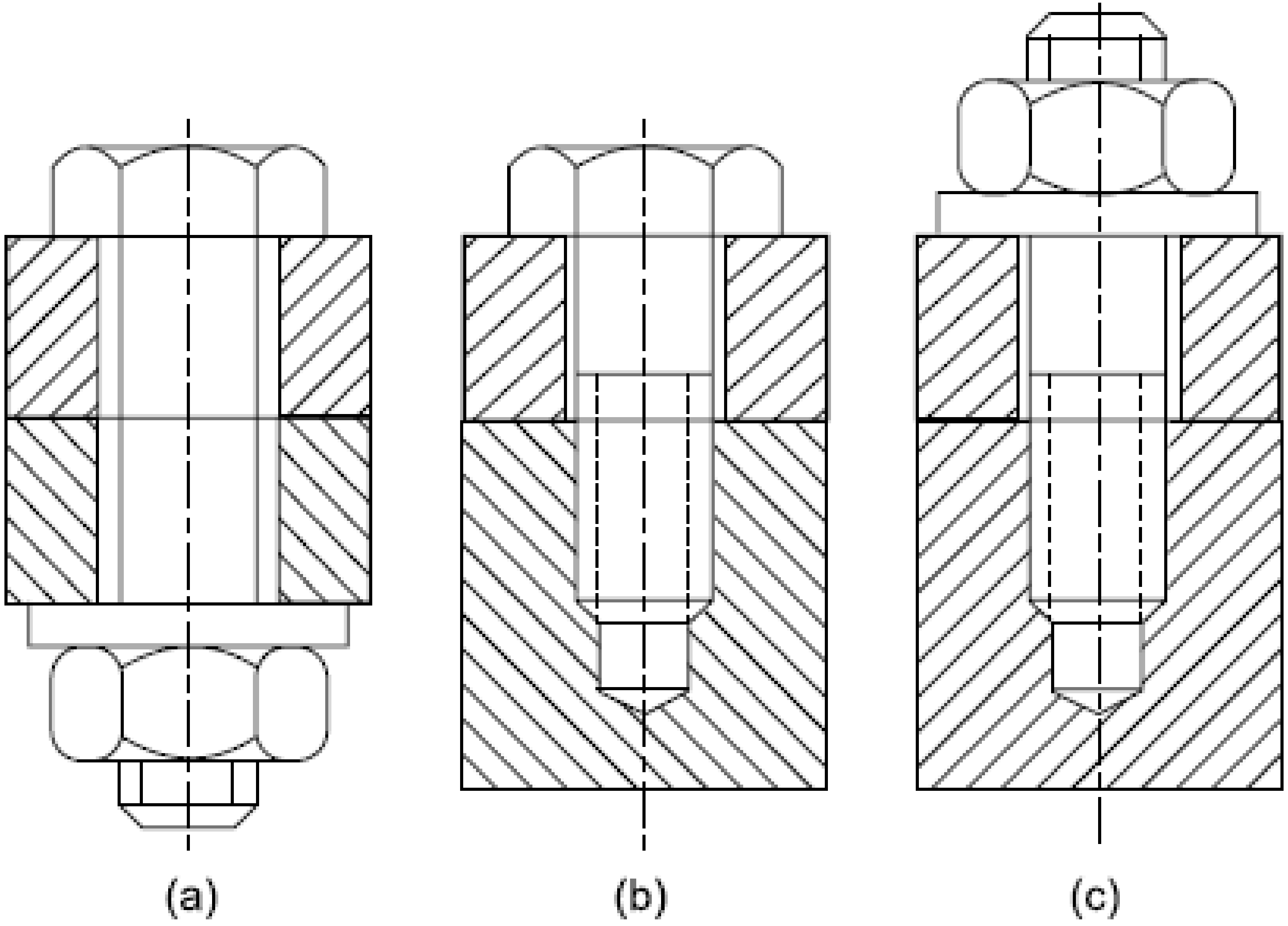


Acme Threads



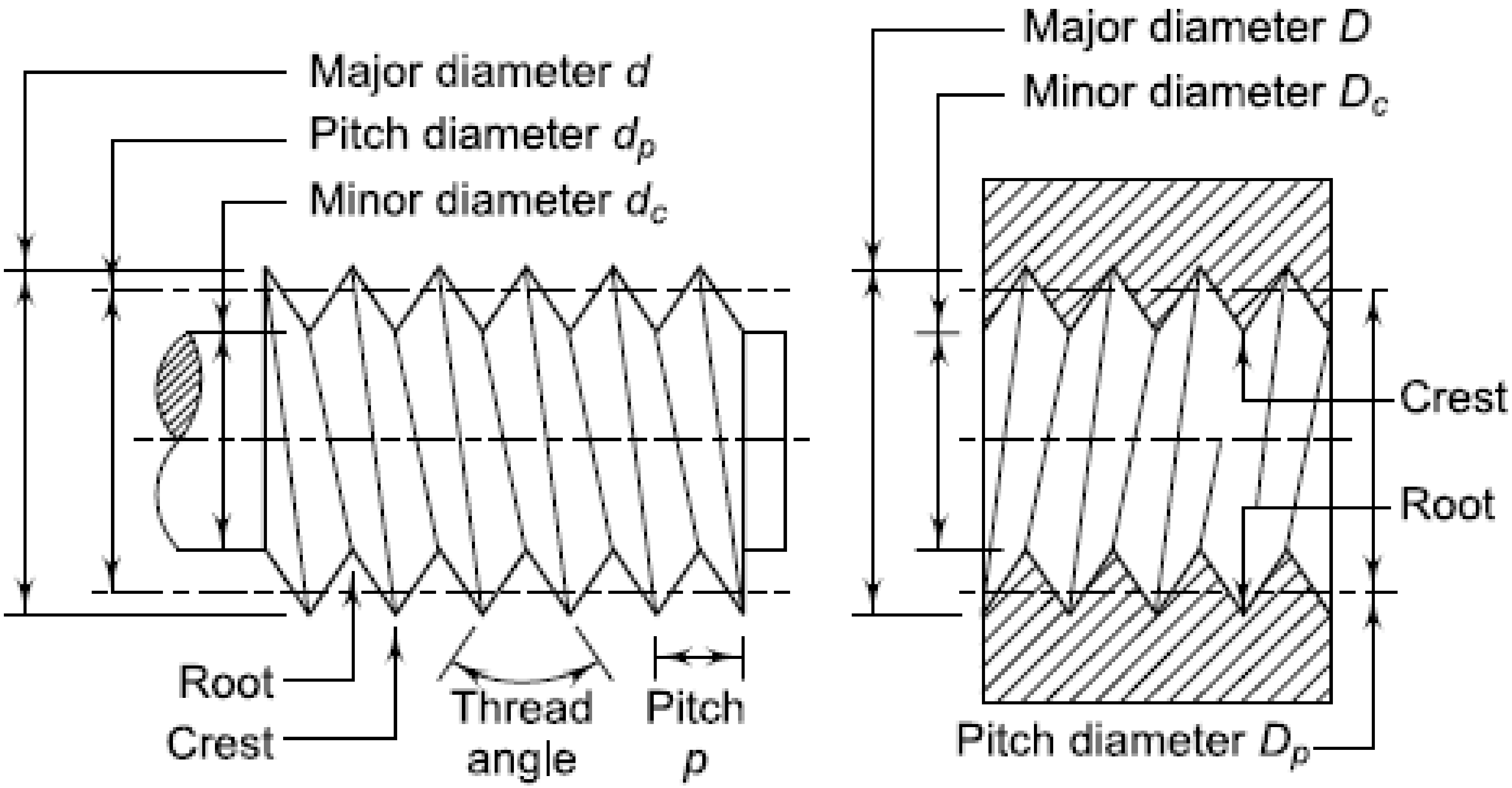
Buttress Threads

Basic Types of Screw Fastenings



Types of Screw Fastening: (a) Through Bolt (b) Tap Bolt (c) Stud

Terminology of Screw Threads



(i) Major Diameter: The major diameter is the diameter of an imaginary cylinder that bounds the crest of an external thread (*d) or the root of* an internal thread (*D*). *The major diameter is the* largest diameter of the screw thread. It is also called the *nominal diameter of the thread*.

(ii) Minor Diameter: The minor diameter is the diameter of an imaginary cylinder that bounds the roots of an external thread (*dc) or the crest of* an internal thread (*Dc*). *The minor diameter is the* smallest diameter of the screw thread. It is also called *core or root diameter of the thread*.

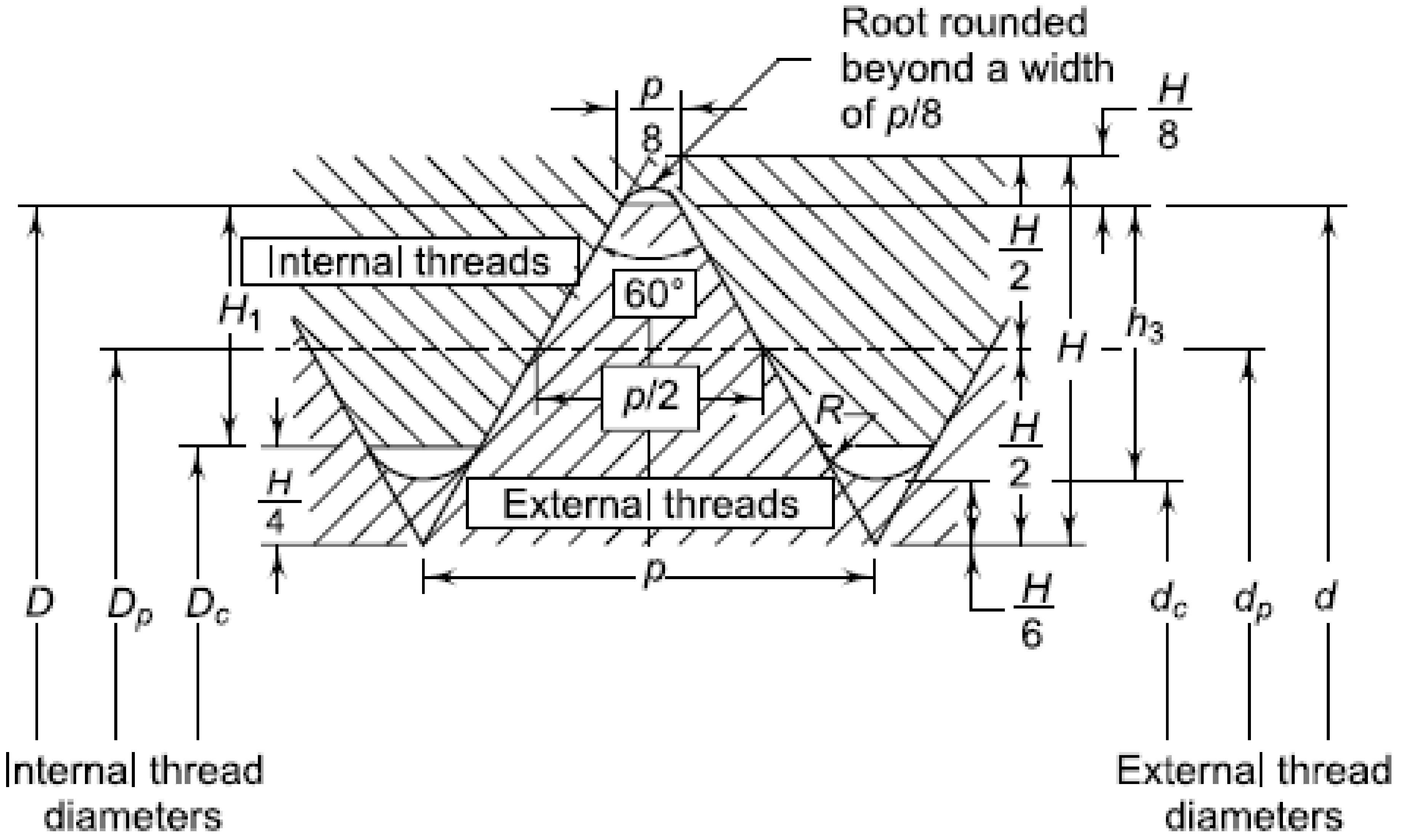
(iii) Pitch Diameter: The pitch diameter is the diameter of an imaginary cylinder, the surface of which would pass through the threads at such points as to make the width of the threads equal to the width of spaces cut by the surface of the cylinder. It is also called the *effective diameter of the thread*. Pitch diameter is denoted by *dp for external threads* and *Dp for internal threads*.

(iv) Pitch: Pitch is the distance between two similar points on adjacent threads measured parallel to the axis of the thread. It is denoted by the letter *p*.

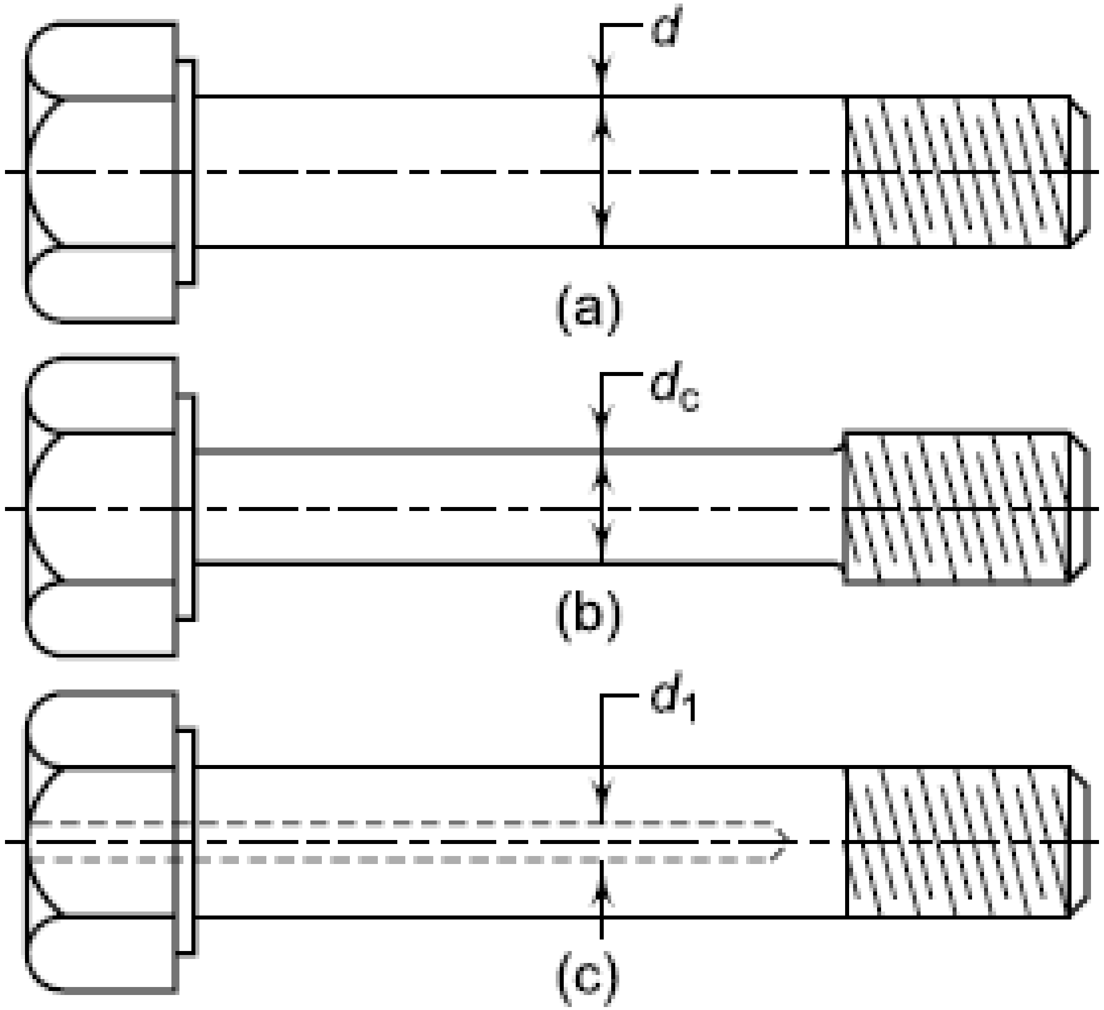
(v) Lead: Lead is the distance that the nut moves parallel to the axis of the screw, when the nut is given one turn.

(vi) Thread Angle: Thread angle is the angle included between the sides of the thread measured in an axial plane. Thread angle is 60o for ISO metric threads.

ISO Metric Screw Threads



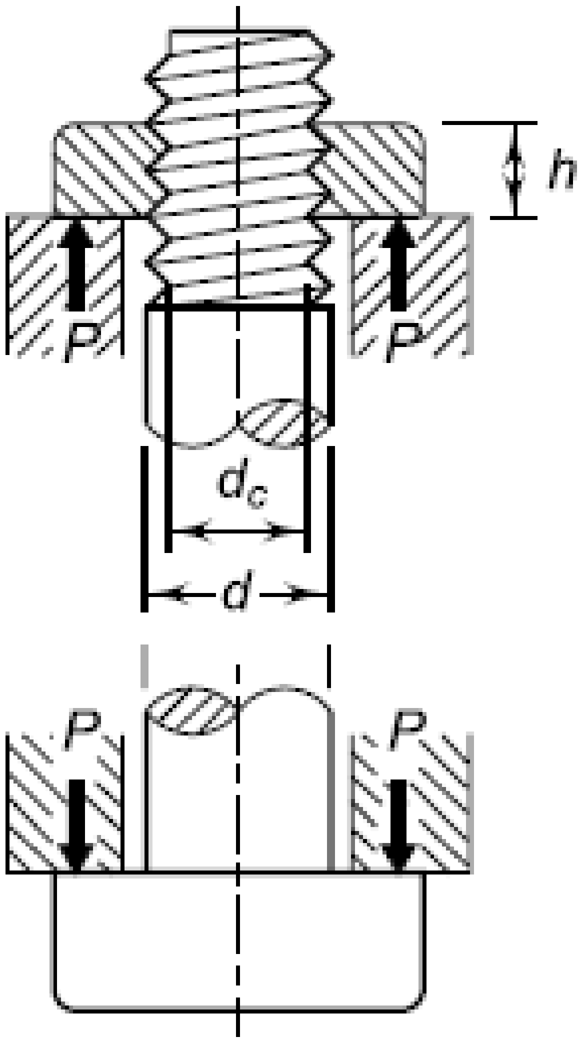
Bolt of Uniform Strength



Bolt Materials

- Lightly loaded small bolts, studs and nuts are made of free cutting steels.
- High strength bolts often fail in fatigue. They are made of plain carbon steels like 40C8 or 45C8 or alloy steels like 35Mn6Mo3, 40Cr4Mo2, 40Ni14 or 40Ni10Cr3Mo6.
- Stainless steel is used for threaded fastener where corrosion resistance is required.

Bolt in Tension



A bolted joint subjected to tensile force *P is shown* in Figure aside. The cross-section at the core diameter *dc is the weakest section. The maximum tensile* stress in the bolt at this cross-section is given by,

$$\sigma_t = \frac{P}{\left(\frac{\pi}{4} d_c^2\right)}$$

The following approximate relationship can be used, for determining the nominal diameter

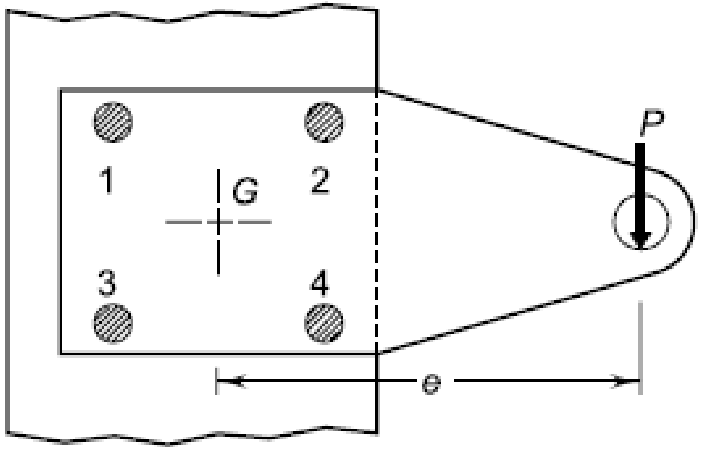
$$d_c = 0.8d$$

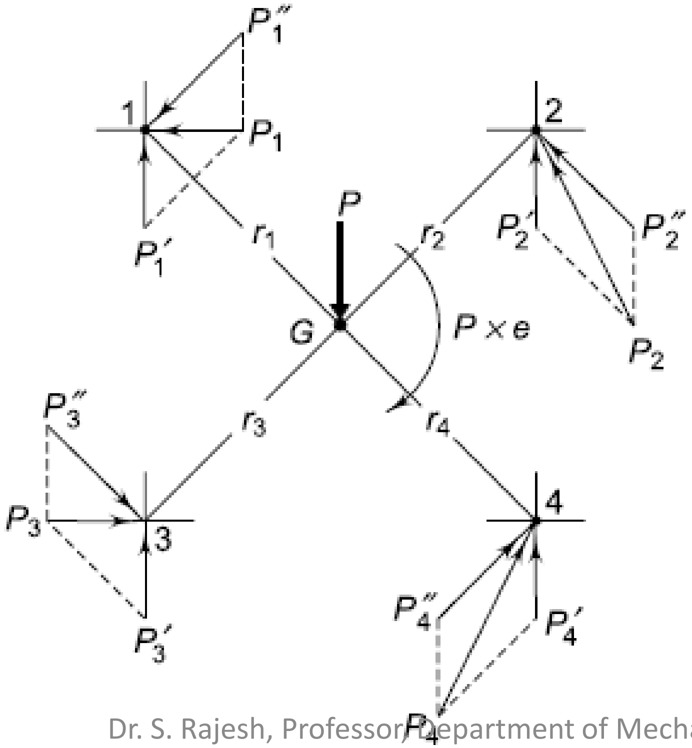
Design of eccentrically loaded threaded joints

There are many applications of the bolted joints which are subjected to eccentric loading such as a wall bracket, pillar crane, etc. The eccentric load may be

- **ECCENTRICALLY LOADED BOLTED JOINTS IN SHEAR**
- **ECCENTRIC LOAD PERPENDICULAR TO AXIS OF BOLT**
- **ECCENTRIC LOAD PARALLEL TO AXIS OF BOLT**
- **ECCENTRIC LOAD ON CIRCULAR BASE**

Eccentrically loaded bolts in shear





$$P_1' = P_2' = P_3' = P_4' = \frac{P}{(\text{No. of bolts})}$$

$$P \times e = P_1'' r_1 + P_2'' r_2 + P_3'' r_3 + P_4'' r_4$$

$$P_1'' = C r_1$$
$$P_2'' = C r_2$$
$$P_3'' = C r_3$$
$$P_4'' = C r_4$$

$$C = \frac{Pe}{(r_1^2 + r_2^2 + r_3^2 + r_4^2)}$$

$$P_1'' = \frac{Per_1}{(r_1^2 + r_2^2 + r_3^2 + r_4^2)}$$

$$P_2'' = \frac{Per_2}{(r_1^2 + r_2^2 + r_3^2 + r_4^2)}$$

$$P_2 = \sqrt{(P_2'^2 + P_2''^2 + 2P_2' P_2'' \cos\theta)}$$

$$\tau = \frac{P_2}{A}$$

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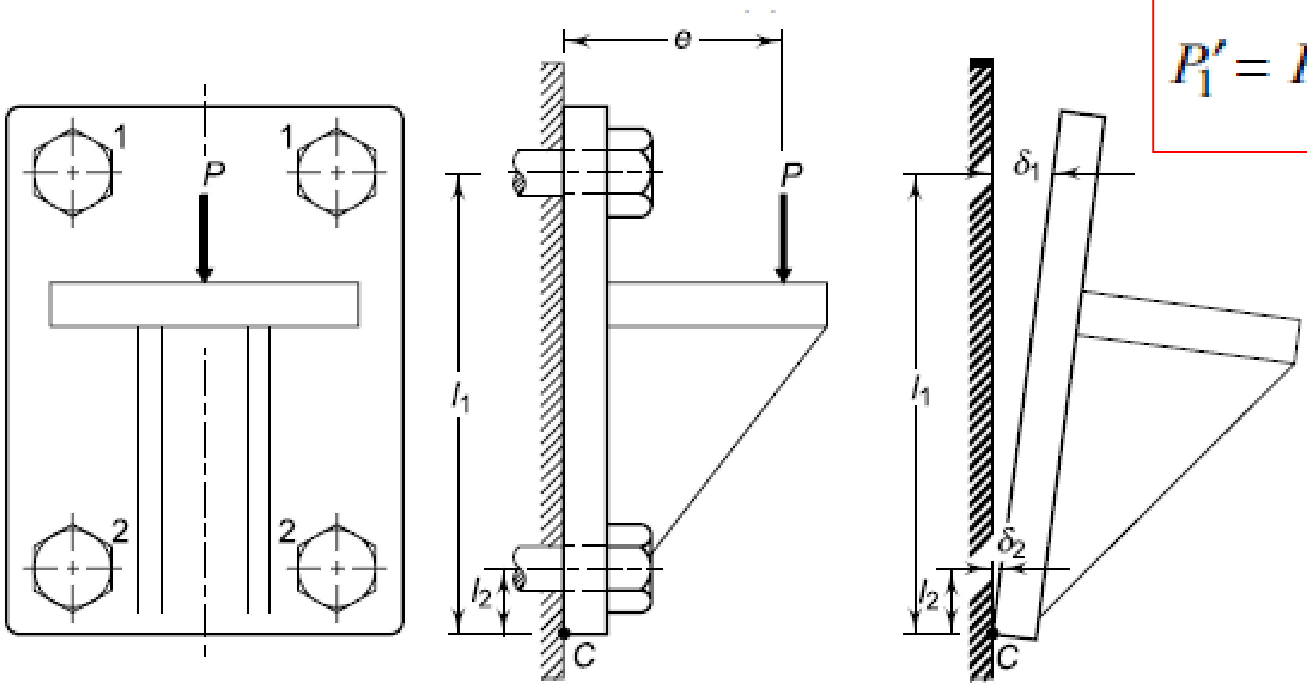
Eccentric load perpendicular to axis of bolt



(a) (b)

$$P_1' = P_2' = \frac{P}{(\text{No. of bolts})}$$

$$Pe = 2P_1''l_1 + 2P_2''l_2$$

$$P_1'' = Cl_1$$
$$P_2'' = Cl_2$$

$$C = \frac{Pe}{2(l_1^2 + l_2^2)}$$

$$P_1'' = \frac{Pel_1}{2(l_1^2 + l_2^2)}$$

$$P_2'' = \frac{Pel_2}{2(l_1^2 + l_2^2)}$$

The direct shear stress in the bolt is given by,

$$\tau = \frac{P_1'}{A}$$

The tensile stress in the bolt is given by, $\sigma_t = \dfrac{P_1''}{A}$

The bolts can be designed on the basis of principal stress theory or principal shear stress theory.

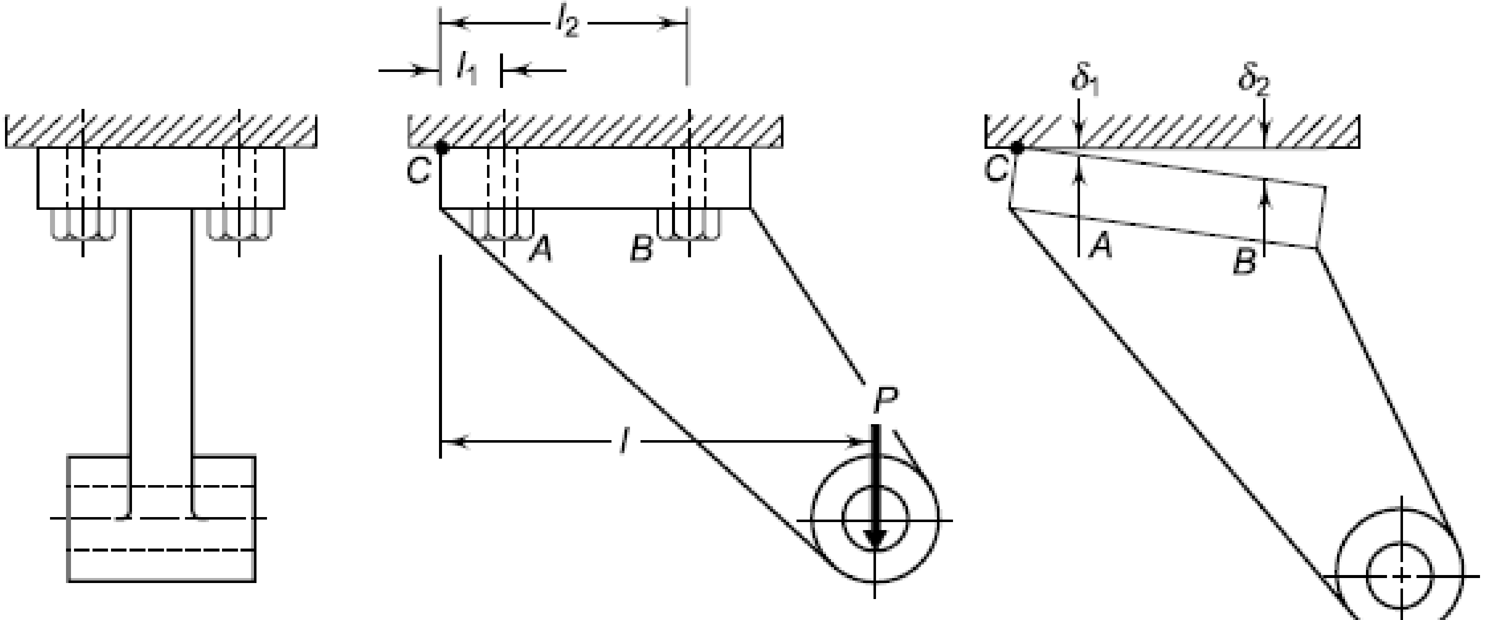
The principal stress *is given by,*

$$\sigma_1 = \frac{\sigma_t}{2} + \sqrt{\left(\frac{\sigma_t}{2}\right)^2 + \tau^2}$$

The principal shear stress *is given by,*

$$\tau_{\max.} = \sqrt{\left(\frac{\sigma_t}{2}\right)^2 + \tau^2}$$

Eccentric load parallel to axis of bolt



(a) (b)

$$P_1' = P_2' = \frac{P}{(\text{No. of bolts})}$$

$$P_1'' = C\,l_1 \qquad P_2'' = Cl_2$$

$$Pl = 2P_1''l_1 + 2P_2''l_2 = 2(Cl_1)\,l_1 + 2(Cl_2)\,l_2$$

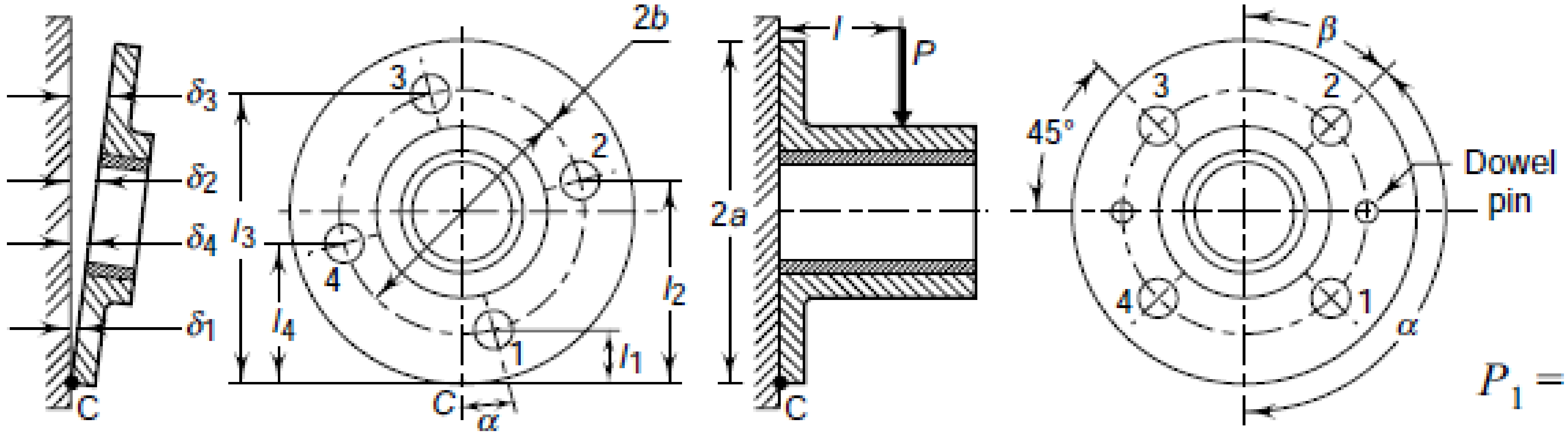
$$C = \frac{Pl}{2(l_1^2 + l_2^2)} \qquad P_2'' = \frac{Pll_2}{2(l_1^2 + l_2^2)}$$

The total tensile force on the bolts is given by, $P_2 = P_2' + P_2''$

Size of the bolts can be determined by using, $\sigma_t = \dfrac{P_2}{A}$ **Where A is given by** $A = \dfrac{\pi}{4} d_c^2$

Note: In general, a bolt, which is located at the farthest distance from the tilting edge, will be subjected to maximum force.

Eccentric load on circular base



(a) (b) (c)

$$P_1 = \frac{Pl(a - b\cos\alpha)}{2(2a^2 + b^2)} = \frac{2Pl(a - b\cos\alpha)}{4(2a^2 + b^2)}$$

$$P_1 = \frac{2\,Pl(a - b\cos\alpha)}{n(2a^2 + b^2)}$$

$$P_{max.} = \frac{2Pl(a + b)}{n(2a^2 + b^2)}$$

If *P1, P2 … are the resisting forces induced in the* bolts, $P_1 \propto l_1$

$$P_1 = Cl_1 \quad P_2 = Cl_2 \quad P_3 = Cl_3 \quad P_4 = Cl_4$$

$$Pl = P_1 l_1 + P_2 l_2 + P_3 l_3 + P_4\, l_4$$

$$Pl = C\ (l_1^2 + l_2^2 + l_3^2 + l_4^2)$$

$$C = \frac{Pl}{(l_1^2 + l_2^2 + l_3^2 + l_4^2)}$$

$$P_1 = \frac{Pl\,l_1}{(l_1^2 + l_2^2 + l_3^2 + l_4^2)}$$

$$l_1 = a - b\cos\alpha$$

$$l_2 = a + b\sin\alpha$$

$$l_3 = a + b\cos\alpha$$

$$l_4 = a - b\sin\alpha$$

$$\boxed{P_1 = \frac{2Pl\left[a + b\cos\left(\frac{180}{n}\right)\right]}{n(2a^2 + b^2)}}$$

$$\boxed{\frac{\pi}{4} d_c^2 (\sigma_t)_{max.} = P_1}$$

Design of Welded Joints

Welded Joints

***Welding can be defined as a process of joining metallic parts by heating to a suitable temperature with or without the application of pressure*. Welding is an economical and efficient method for obtaining a permanent joint of metallic parts.**

Welded joints offer the following advantages compared with riveted joints:

i. **Riveted joints require additional cover plates, gusset plates, straps, clip angles and a large number of rivets, which increase the weight. Since there are no such additional parts, welded assembly results in lightweight construction.**
ii. **Due to the elimination of these components, the cost of welded assembly is lower than that of riveted joints.**
iii. **The design of welded assemblies can be easily and economically modified to meet the changing product requirements. Alterations and additions can be easily made in the existing structure by welding.**

iv. **Welded assemblies are tight and leakproof as compared with riveted assemblies.**
v. **The production time is less for welded assemblies.**
vi. **When two parts are joined by the riveting method, holes are drilled in the parts to accommodate the rivets. The holes reduce the cross-sectional area of the members and result in stress concentration. There is no such problem in welded connections.**
vii. **A welded structure has smooth and pleasant appearance. The projection of rivet head adversely affects the appearance of the riveted structure.**
viii. **The strength of welded joint is high. Very often, the strength of the weld is more than the strength of the plates that are joined together.**
ix. **Machine components of certain shape, such as circular steel pipes, find difficulty in riveting. However, they can be easily welded.**

Welded joints have the following disadvantages:

i. **As compared with cast iron structures, the capacity of welded structure to damp vibrations is poor.**
ii. **Welding results in a thermal distortion of the parts, thereby inducing residual stresses. In many cases, stress-relieving heat treatment is required to relieve residual stresses. Riveted or cast structures do not require such stress relieving treatment.**
iii. **The quality and the strength of the welded joint depend upon the skill of the welder. It is difficult to control the quality when a number of welders are involved.**
iv. **The inspection of the welded joint is more specialised and costly compared with the inspection of riveted or cast structures.**

The advantages have made welding suitable for joining components in various machines and structures. Some typically welded machine components are listed below.

- **Pressure vessels, steel structures.**
- **Flanges welded to shafts and axles.**
- **Crank shafts**
- **Heavy hydraulic turbine shafts**
- **Large gears, pulleys, flywheels**
- **Gear housing**
- **Machine frames and bases**
- **Housing and mill-stands.**

Welding Processes

Welding processes are broadly classified into the following two groups:

(1) Welding processes that use heat alone to join the two parts.

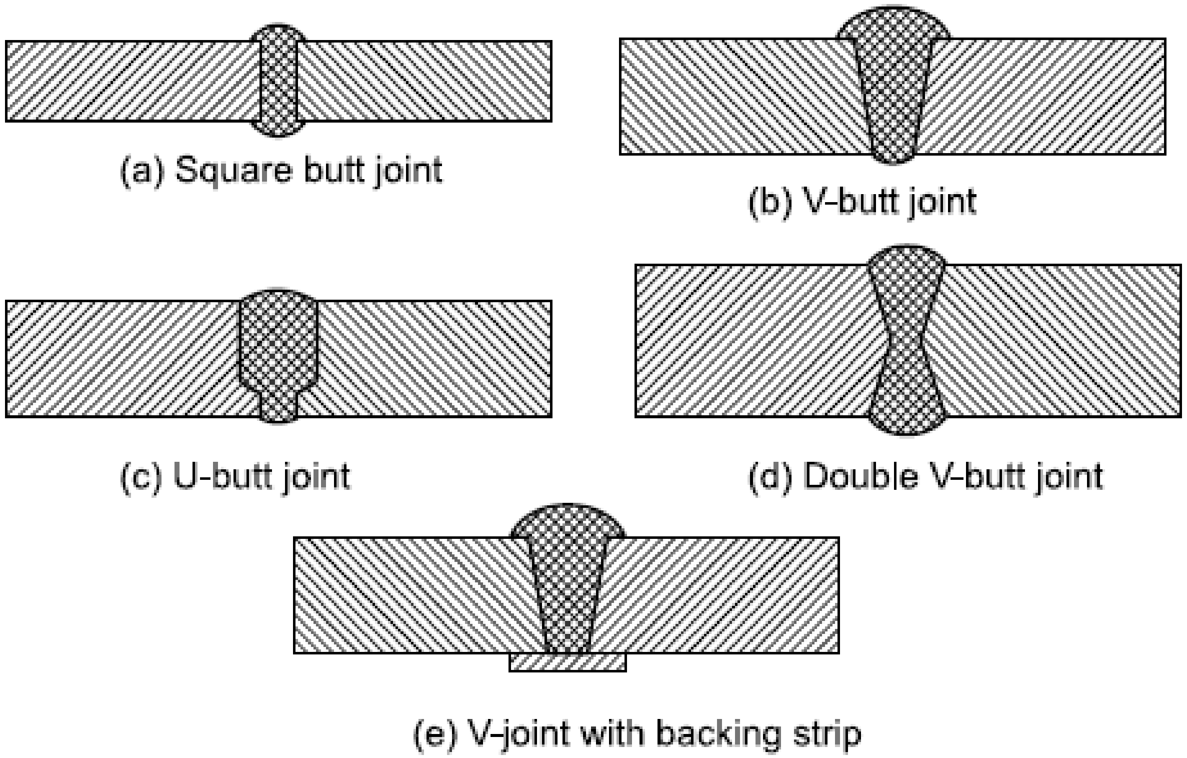
- The welding process that uses heat alone is called the *fusion welding process*.
- *In this method*, the parts to be joined are held in position and molten metal is supplied to the joint.
- Fusion welding is further classified into the following three groups:
  i. Thermit welding
  ii. Gas welding
  iii. Electric arc welding

(2) Welding processes that use a combination of heat and pressure to join the two parts.

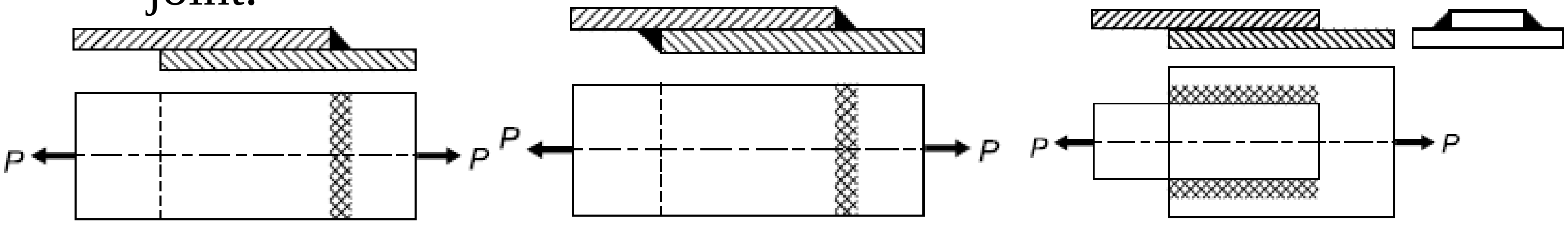
- Welding processes that use a combination of heat and pressure to join the two parts are classified into the following two groups:
  i. Forge welding
  ii. Electric resistance welding

Types of Welded Joints

- Welded joints are divided into two groups—butt joints and fillet joints.
- ***Butt Joints**: **A butt joint can be defined as a joint between two components lying approximately in the same plane**.* A butt joint connects the ends of the two plates.
- The types of butt joints are illustrated in Figure below.

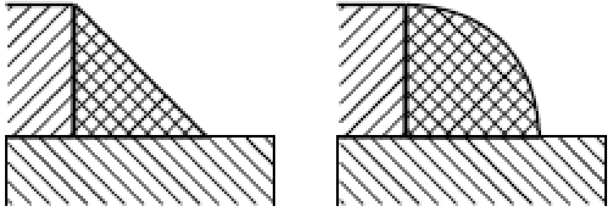


- ***Fillet Joint: A fillet joint, also called a lap joint, is a joint between two overlapping plates or components.*** *A* fillet weld consists of an approximately triangular cross-section joining two surfaces at right angles to each other.
- There are two types of fillet joints—**transverse** and **parallel**, as shown in Figure below.
- A fillet weld is called ***transverse***, *if the direction of the* weld is perpendicular to the direction of the force acting on the joint.
- A fillet weld is called ***parallel* or *longitudinal***, *if the direction of weld is parallel* to the direction of the force acting on the joint.

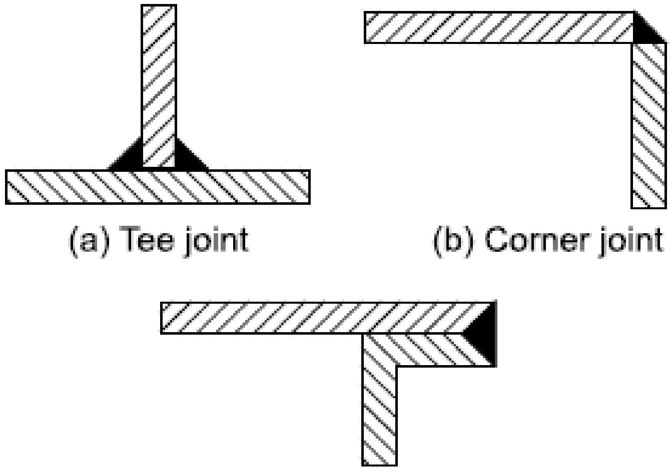


(a) Single transverse fillet weld (b) Double transverse fillet welds (c) Double parallel fillet welds

- There are two types of cross-sections for fillet weld—normal and convex.
- The *normal weld consists of an isosceles triangle—a* triangle having two equal sides.
- *A* convex weld requires more filler material and more labour. There is more stress concentration in a convex weld compared to a triangular weld. Therefore, normal weld is preferred over convex weld.

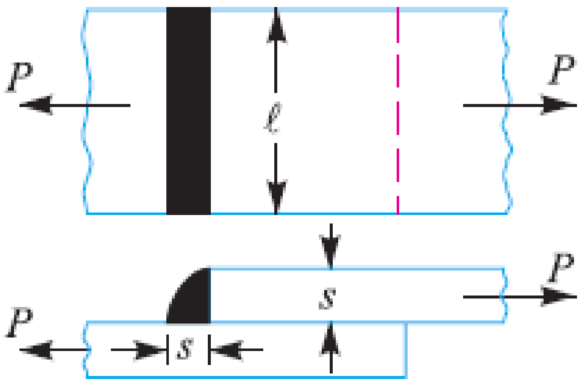

(a) Normal (b) Convex

- **Tee-Joint:** A *tee-joint is a joint between* two components located at right angles to each other in the form of a T.
- **Corner Joint:** A *corner joint is a joint between two* components, which are at right angles to each other in the form of an angle.
- **Edge Joint:** An *edge joint is a joint between the* edges of two or more parallel components

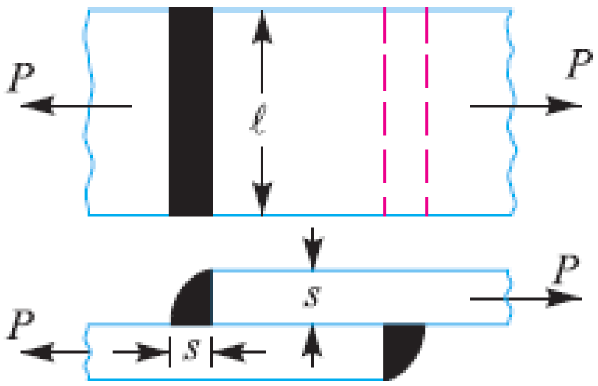

(a) Tee joint (b) Corner joint

(c) Edge joint

Strength of Transverse Fillet Welded Joints



(*a*) Single transverse fillet weld.



(*b*) Double transverse fillet weld.

t = Throat thickness (BD),

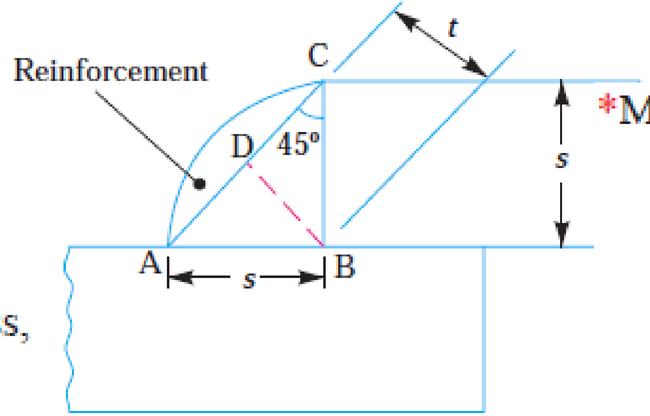
s = Leg or size of weld,

= Thickness of plate, and

l = Length of weld,

we find that the throat thickness,

$t = s \times \sin 45° = 0.707\, s$



*Minimum area of the weld or throat area,

A = Throat thickness × Length of weld

$= t \times l = 0.707\, s \times l$

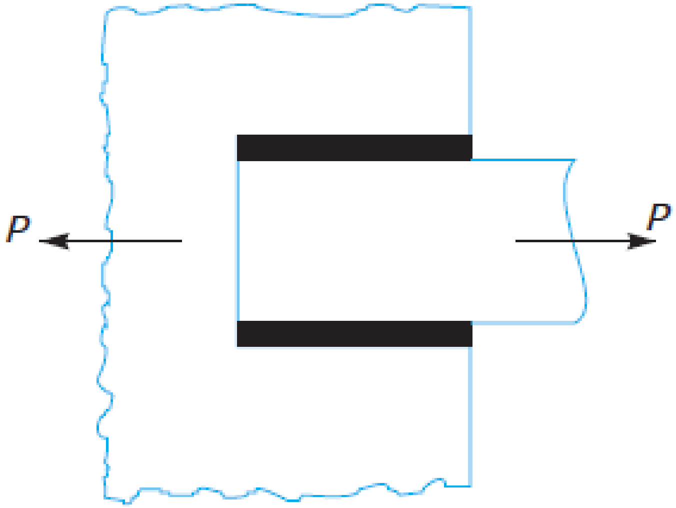
If σ_t is the allowable tensile stress for the weld metal, then the tensile strength of the joint for single fillet weld,

$$P = \text{Throat area} \times \text{Allowable tensile stress} = 0.707\, s \times l \times \sigma_t$$

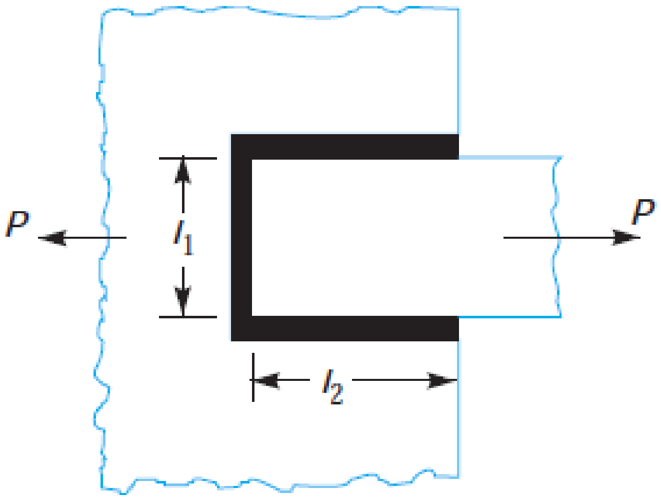
and tensile strength of the joint for double fillet weld,

$$P = 2 \times 0.707\, s \times l \times \sigma_t = 1.414\, s \times l \times \sigma_t$$

Strength of Parallel Fillet Welded Joints



(*a*) Double parallel fillet weld.



(*b*) Combination of transverse and parallel fillet weld.

If τ is the allowable shear stress for the weld metal, then the shear strength of the joint for single parallel fillet weld,

$$P = \text{Throat area} \times \text{Allowable shear stress} = 0.707\, s \times l \times \tau$$

and shear strength of the joint for double parallel fillet weld,

$$P = 2 \times 0.707 \times s \times l \times \tau = 1.414\, s \times l \times \tau$$

If there is a combination of single transverse and double parallel fillet welds as shown in Fig. (b), then the strength of the joint is given by the sum of strengths of single transverse and double parallel fillet welds. Mathematically,

$$P = 0.707 s \times l_1 \times \sigma_t + 1.414\, s \times l_2 \times \tau$$

Circular Fillet weld subjected to Torsion

d = Diameter of rod,

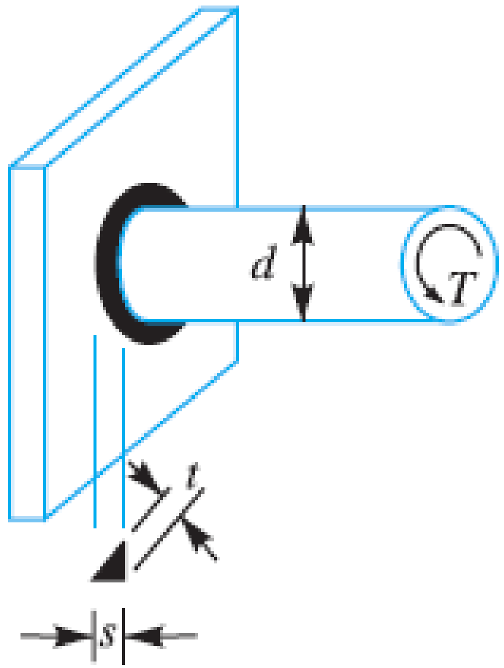
r = Radius of rod,

T = Torque acting on the rod,

s = Size (or leg) of weld,

t = Throat thickness,

*J = Polar moment of inertia of the weld section $= \dfrac{\pi t d^3}{4}$



We know that shear stress for the material,

$$\tau = \frac{T.r}{J} = \frac{T \times d/2}{J}$$

$$= \frac{T \times d/2}{\pi t d^3/4} = \frac{2T}{\pi t d^2}$$

This shear stress occurs in a horizontal plane along a leg of the fillet weld. The maximum shear occurs on the throat of weld which is inclined at 45° to the horizontal plane.

$\therefore$ Length of throat, $t = s \sin 45° = 0.707\, s$

$$\tau_{max} = \frac{2T}{\pi \times 0.707\, s \times d^2} = \frac{2.83\, T}{\pi\, s\, d^2}$$

Long Fillet weld subjected to Torsion

Let T = Torque acting on the vertical plate,

l = Length of weld,

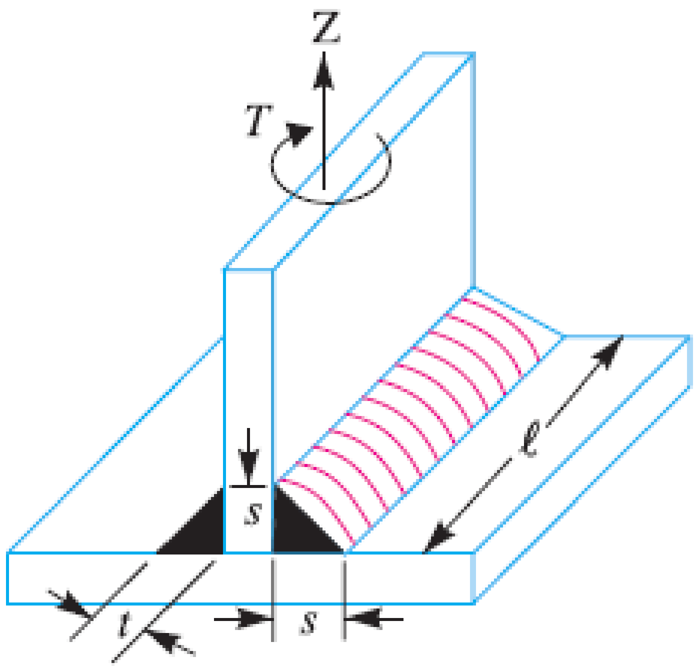
s = Size (or leg) of weld,

t = Throat thickness, and

J = Polar moment of inertia of the weld section

$$= 2 \times \frac{t \times l^3}{12} = \frac{t \times l^3}{6} \quad \ldots$$

($\because$ of both sides weld)

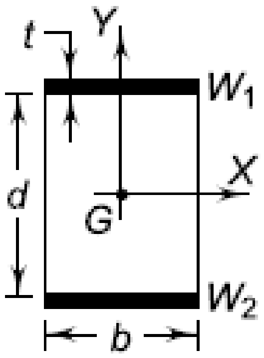


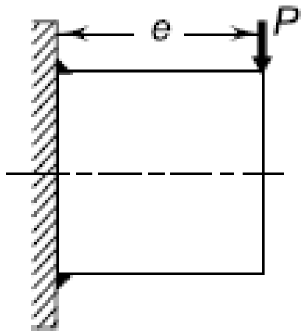
$$\therefore \text{ Shear stress,} \qquad \tau = \frac{T \times l/2}{t \times l^3/6} = \frac{3\,T}{t \times l^2}$$

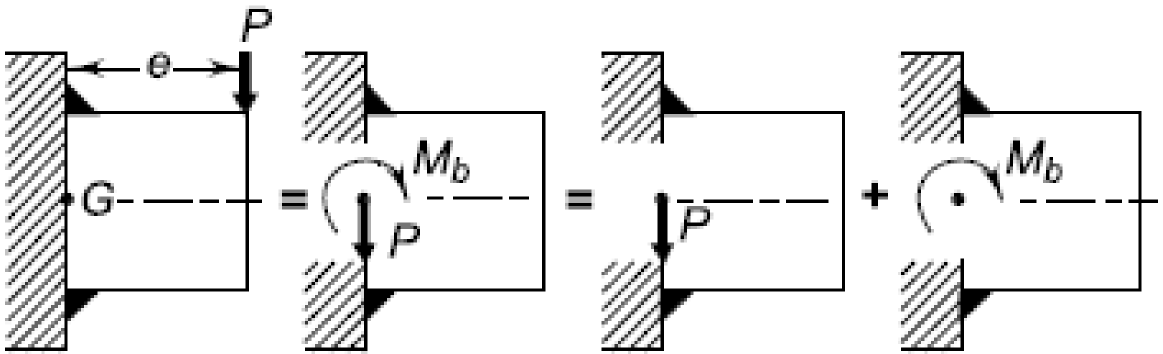
The maximum shear stress occurs at the throat and is given by

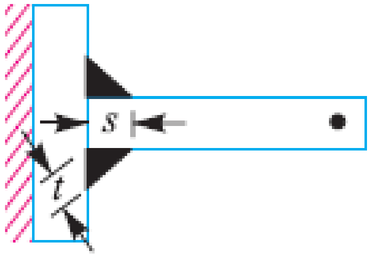
$$\tau_{max} = \frac{3T}{0.707\,s \times l^2} = \frac{4.242\,T}{s \times l^2}$$

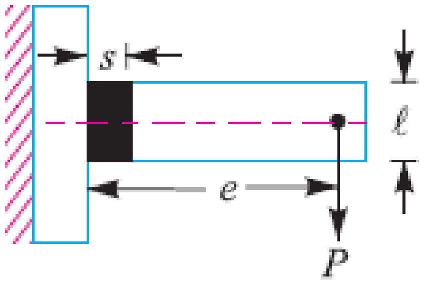
Welded Joint subjected to Bending Moment











$$\tau_1 = \frac{P}{A} \qquad \sigma_b = \frac{M_b y}{I}$$

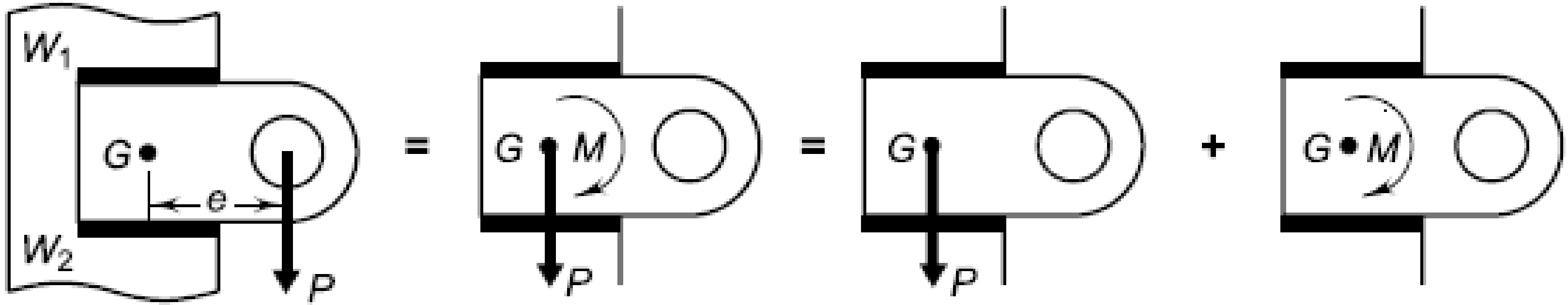
where

I = moment of inertia of all welds based on the throat area

y = distance of the point in weld from the neutral-axis

$$\tau = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + (\tau_1)^2}$$

Eccentric Load in the Plane of Welds



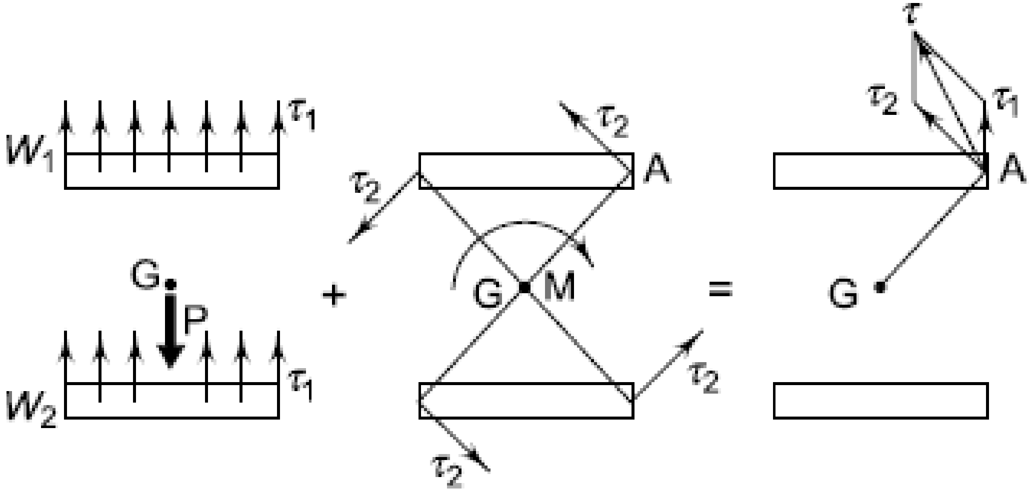
where

r = distance of a point in the weld from G

J = polar moment of inertia of all welds about G

$$\tau_1 = \frac{P}{A}$$

$$\tau_2 = \frac{Mr}{J}$$



$$\tau_A = \sqrt{(\tau_1)^2 + (\tau_2)^2 + 2\tau_1 \times \tau_2 \times \cos\theta}$$

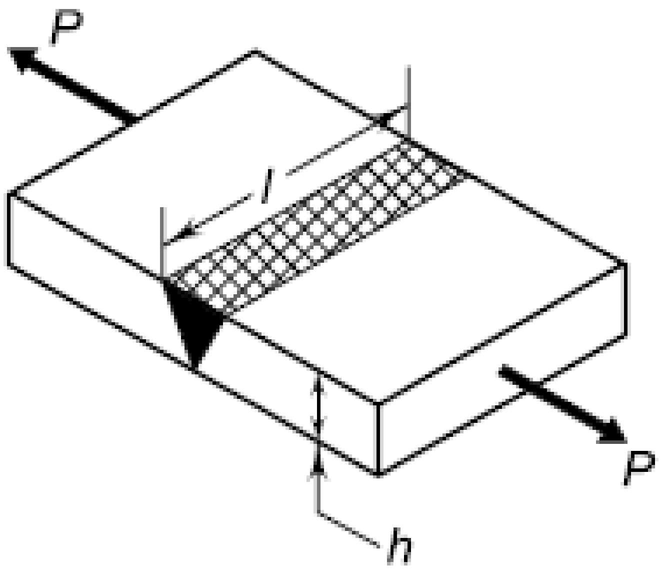
Strength of Butt Welds

A butt welded joint, subjected to tensile force P, is shown in Figure below. The average tensile stress in the weld is given by,

$$\sigma_t = \frac{P}{hl}$$

where,

σ_t = tensile stress in the weld (N/mm^2)
P = tensile force on the plates (N)
h = throat of the butt weld (mm)
l = length of the weld (mm)



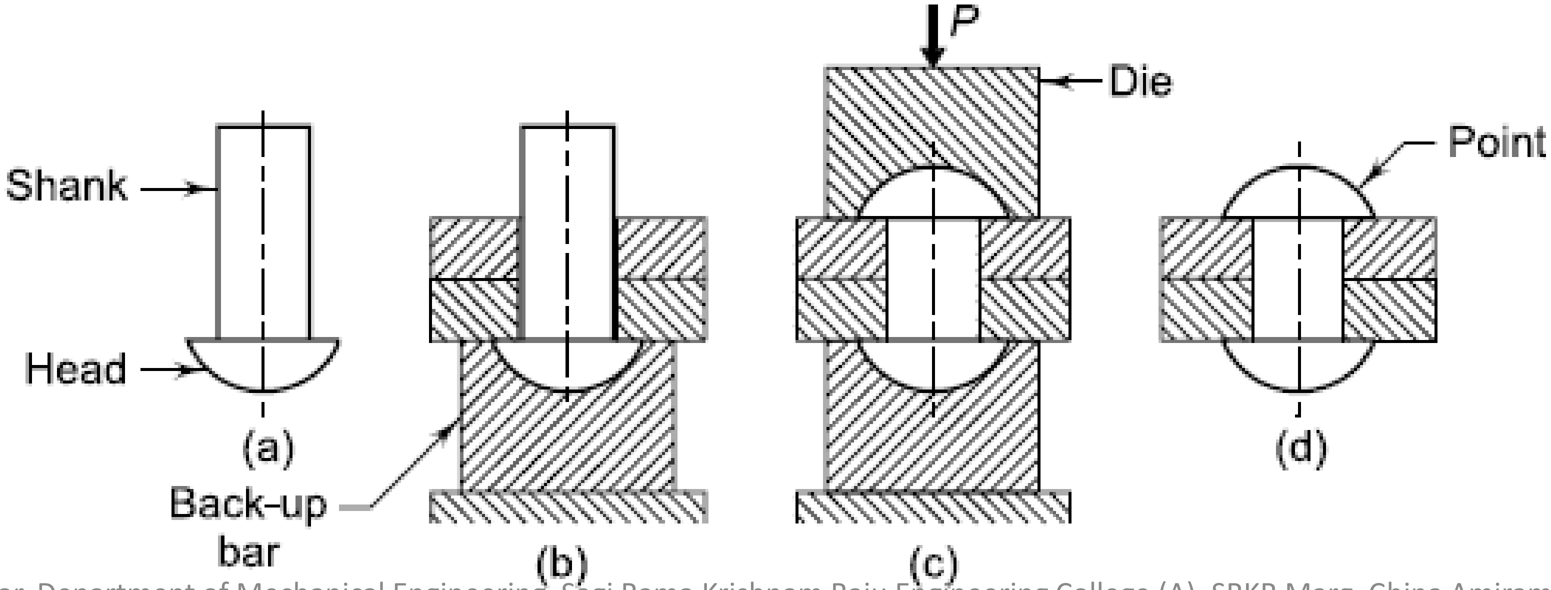
The strength equation of butt joint can be written as,

$$P = \sigma_t \, tl$$

Design of Riveted Joints

Riveted Joints

- A rivet consists of a cylindrical shank with a head at one end as shown in Fig. (a). This head is formed on the shank by an upsetting process in a machine called an *automatic header.* The rivet is inserted in the holes of the parts being assembled as shown in Fig. (b) and the head is firmly held against the back up bar. In the riveting process, the protruding end of the shank is upset by hammer blows to form the closing head. In rivet terminology, the closing head is called the *point.*
- The head, shank and point are three main parts of the rivet.
- A rivet is specified by the shank diameter of the rivet, e.g., a 20 mm rivet means a rivet having 20 mm as the shank diameter. The standard sizes of rivets are 12, 14, 16, 18, 20, 22, 24, 27, 30, 33,36, 39, 42 and 48 mm.
- In the past, riveted joints were widely used for making permanent joints in engineering applications like boilers, pressure vessels, reservoirs, ships, trusses, frames and cranes.



Advantages of Riveted Joints

A riveted joint has the following advantages over a welded joint:

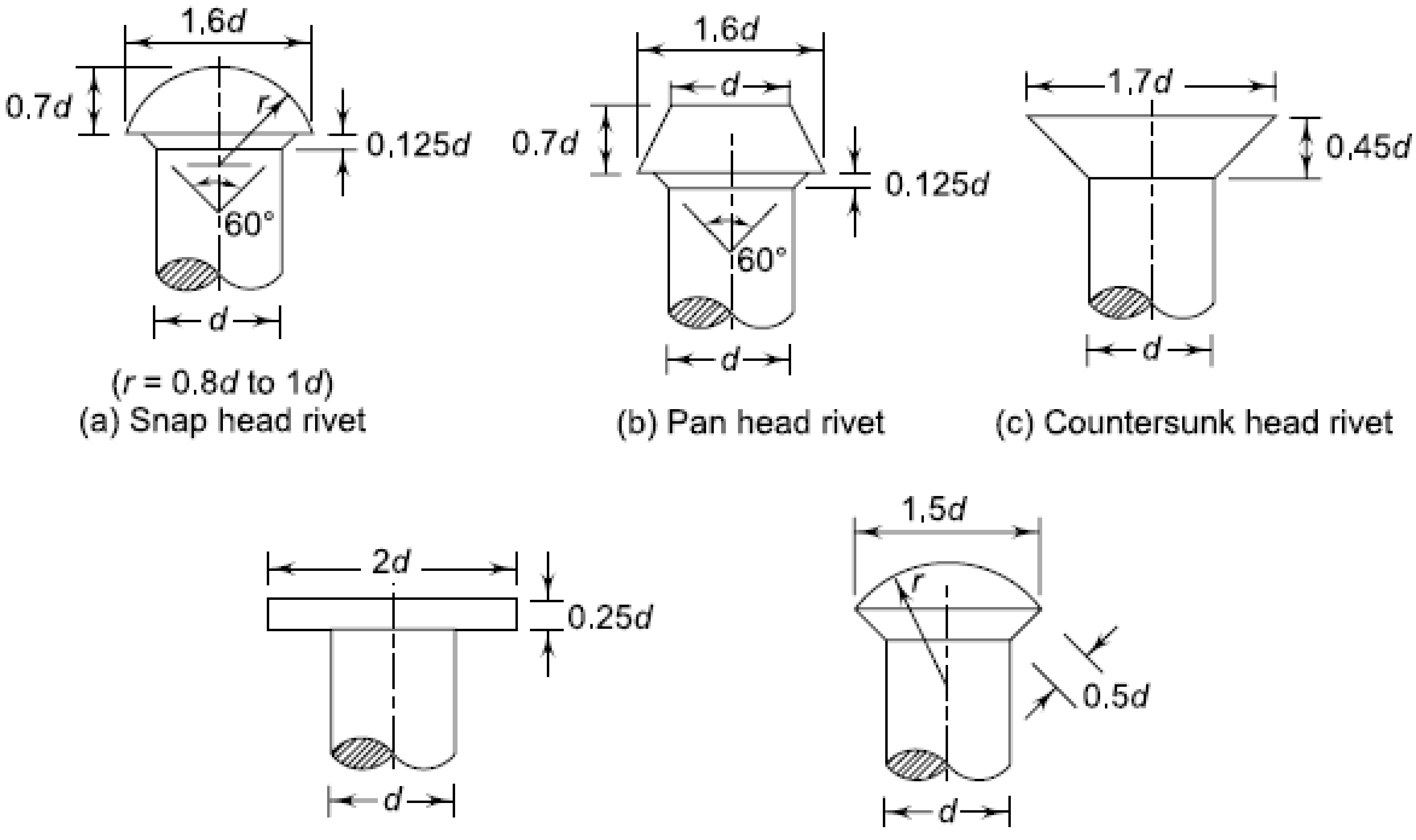
i. **A riveted joint is more reliable than a welded joint in applications which are subjected to vibrations and impact forces.**
ii. **Welded joints are, in general, restricted to steel parts. Riveted joints can be used for non-ferrous metals like aluminium alloy, copper, brass or even non-metals like asbestos or plastics.**
iii. **The heat required for welding causes warping and affects the structure of heat treated components. The parts assembled by riveted joints are free from such thermal after-effects.**
iv. **The quality of riveted joint can be easily checked, while inspection methods for welded joint, such as radiographic inspection of pressure vessels, are costly and time consuming.**
v. **When the riveted joint is dismantled, the connected components are less damaged compared with those of welded joints.**

Disadvantages of Riveted Joints

The disadvantages of riveted joints compared with welded joints are as follows:

i. **The material cost of riveted joints is more than the corresponding material cost of welded joints due to high consumption of metal.**
ii. **The labour cost of riveted joints is more than that of welded joints. Riveted joint requires higher labour input due to necessity to perform additional operations like layout and drilling or punching of holes.**
iii. **The overall cost of riveted joint is more than that of welded joint due to increased metal consumption and higher labour input. On the other hand, welding is cheaper compared with riveting.**
iv. **Riveted assemblies have more weight than welded assemblies due to strap-plates and rivets. Welded assemblies result in lightweight construction.**
v. **Riveting process creates more noise than welding due to hammer blows.**
vi. **Holes required to insert rivets cause stress concentration.**

Types of Rivet Heads



(r = 0.8d to 1d)
(a) Snap head rivet

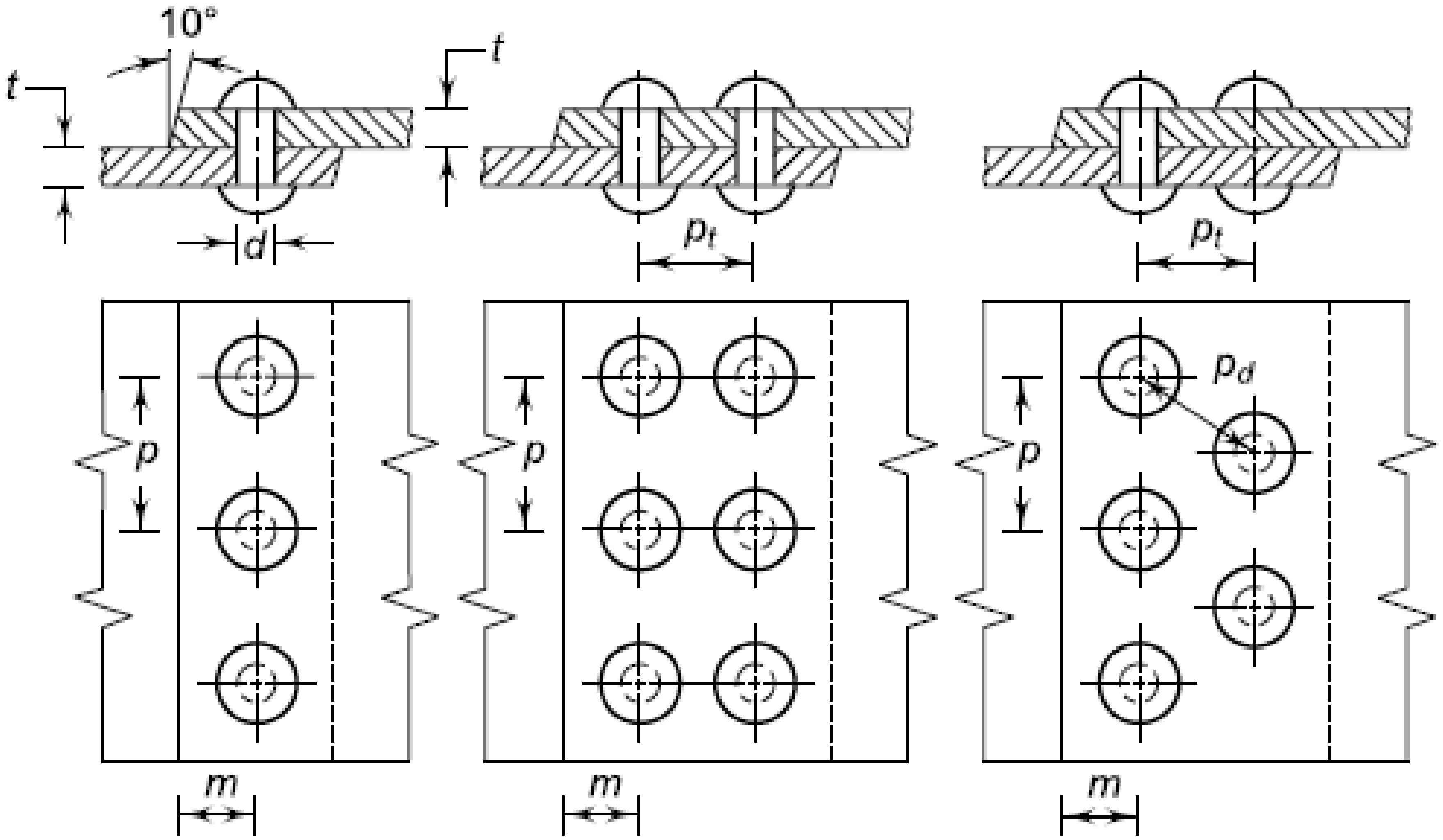
(b) Pan head rivet

(c) Countersunk head rivet

(d) Flat head rivet

(r = 1.5 d)
(e) Half countersunk head rivet

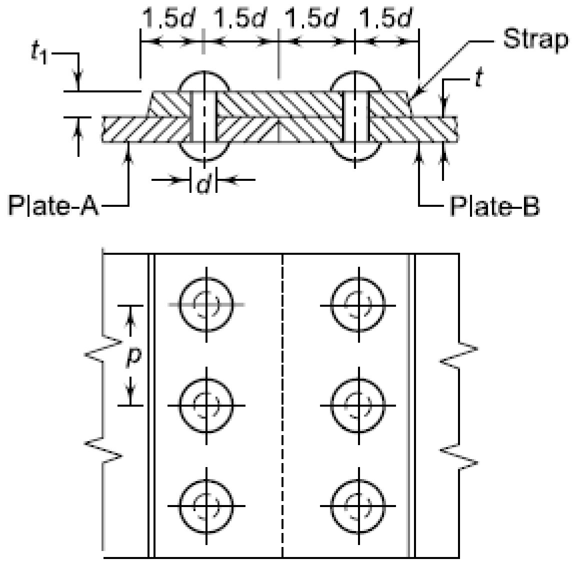
Types of Riveted Joints



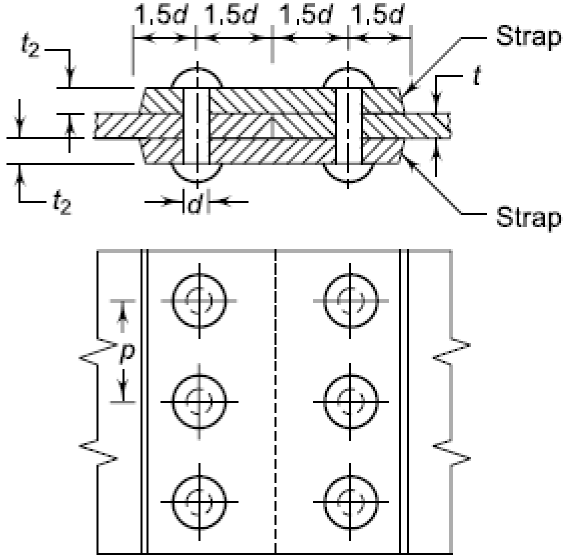
single-riveted lap joint

double-riveted lap joint (chain pattern)

double-riveted lap joint (zig-zag pattern)



Single-riveted Single Strap Butt Joint



Single-riveted Double Strap Butt Joint

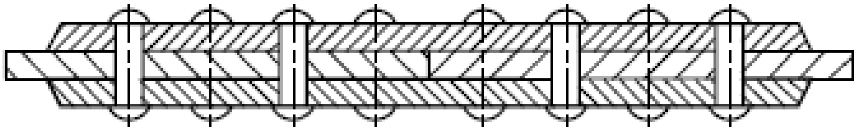
Rivet Materials

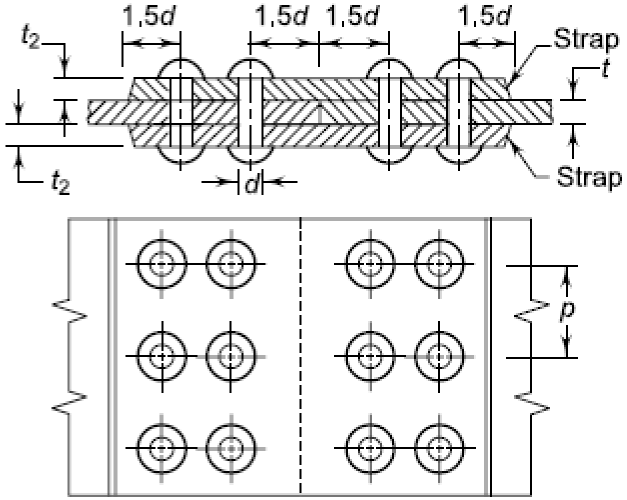
Rivets used in most of the applications are made of mild steel. There are two varieties of steel rivet bars—*hot rolled steel rivet bar and high-tensile steel rivet bar. Their chemical composition is as follows:*

carbon = 0.23% (max)

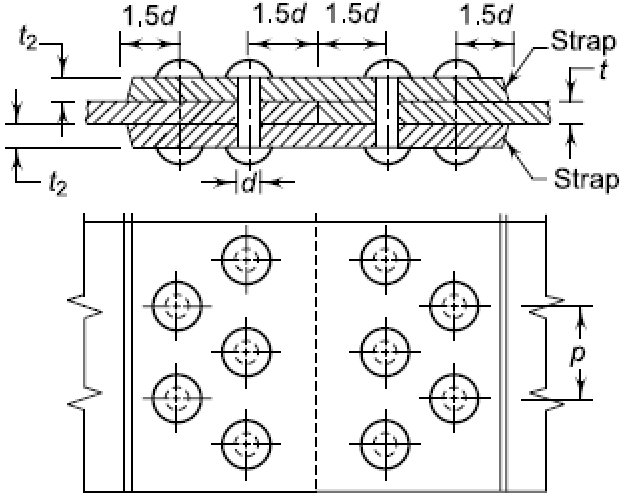
sulphur = 0.05% (max)

phosphorus = 0.05% (max)

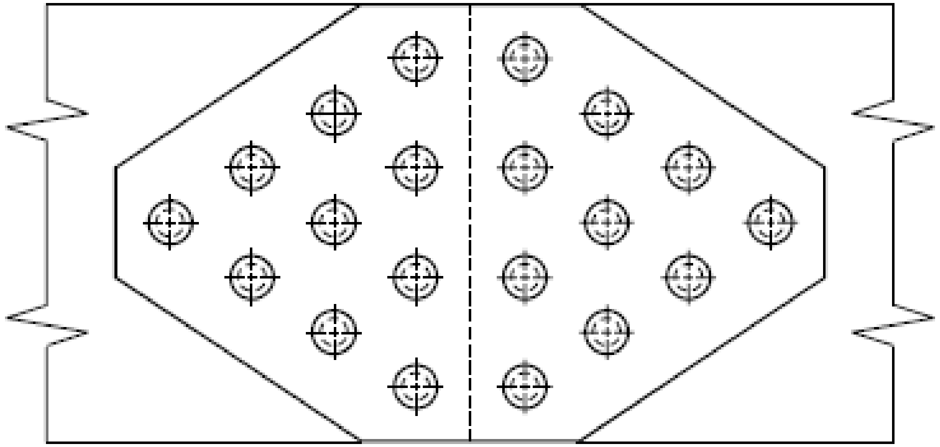




Double-riveted Double Strap Butt Joint (Chain pattern)



Double-riveted Double Strap Butt Joint (zig-zag pattern)



Diamond or Lozenge Joint

Terminology of Riveted Joints

(i) Pitch (p): **The pitch of the rivet is defined as the distance between the centre of one rivet to the centre of the adjacent rivet in the same row. Usually, *$p = 3d$* where *d* is shank diameter of the rivet.**

(ii) Margin (m): **The margin is the distance between the edge of the plate to the centreline of rivets in the nearest row. Usually, *$m = 1.5d$***

(iii) Transverse Pitch (p_t): **Transverse pitch, also called back pitch or row pitch, is the distance between two consecutive rows of rivets in the same plate. Usually,**

$p_t = 0.8p$ ***(for chain riveting)***

$= 0.6p$ ***(for zig-zag riveting)***

(iv) Diagonal Pitch (p_d): **Diagonal pitch is the distance between the centre of one rivet to the centre of the adjacent rivet located in the adjacent row.**

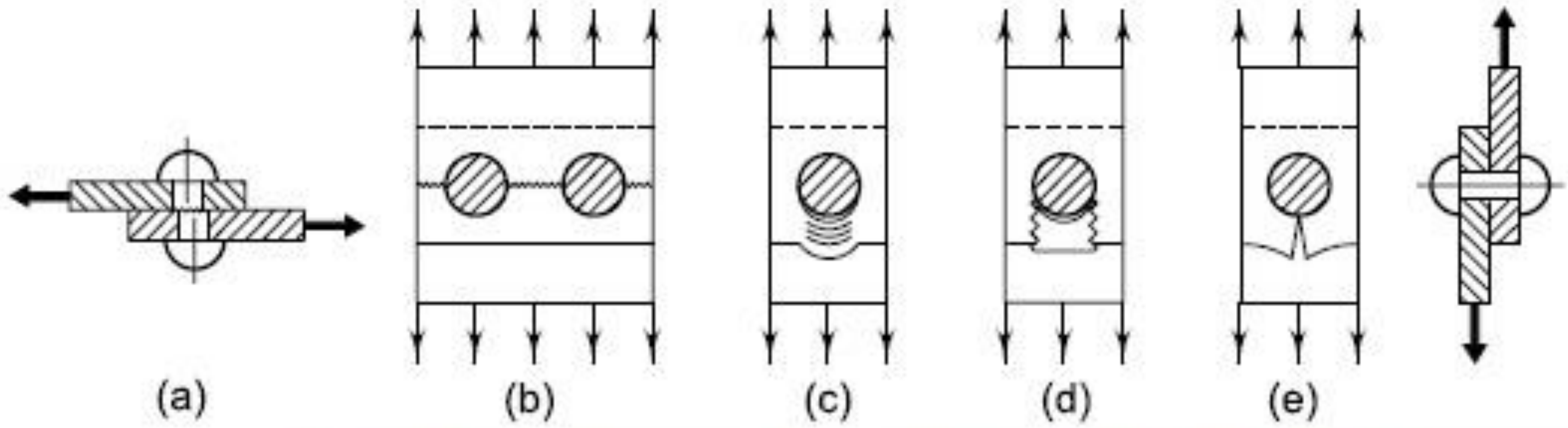
Strength Equations

- ***The strength of riveted joint is defined as the force that the joint can withstand without causing failure.*** **When the operating force acting on the joint exceeds this force, failure occurs. Strength equations can be written for each type of failure.**
- **However, in analysis of riveted joints, mainly three types of failure are considered. They are as follows:**

  i. shear failure of the rivet;

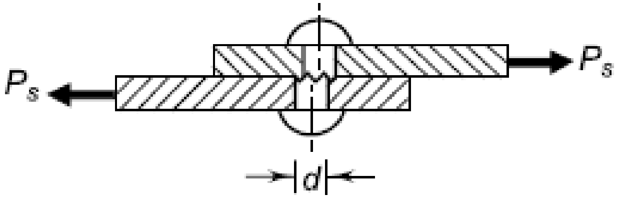
  ii. tensile failure of the plate between rivets; and

  iii. crushing failure of the plate.



Types of Failure in Riveted Joint (a) Shear Failure of Rivet (b) Tensile Failure of Plate between Rivets (c) Crushing Failure of Plate by Rivet (d) Shear Failure of Plate by Rivet (e) Tearing of Margin *(i) Shear Strength of Rivet: The shear failure in* **the rivet of a single-riveted lap joint is illustrated in Figure beow. In this case, the rivet is in single shear. The strength equation is written in the following way,**

$$\boxed{P_s = \frac{\pi}{4} d^2 \tau}$$



Where, P_s = ***shear resistance of rivet per pitch*** **length (N);** ***d = shank diameter of rivet (mm); τ = permissible shear stress for rivet*** **material (N/mm²)**

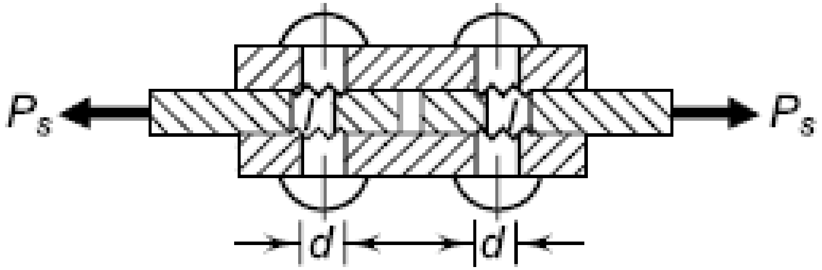
- **In case of double or triple riveted lap joints, there are number of rivets and the above equation is modified and written in the following way:**

$$\boxed{P_s = \frac{\pi}{4} d^2 \tau n \quad \text{(for single shear)}}$$

Where, n = number of rivets per pitch length. For double-riveted joint, n = 2; For triple-riveted joint, n = 3

- **In case of double-strap single-riveted butt joint, the rivets are subjected to double shear as shown in Figure below. The area that resists shear failure is twice the cross-sectional area of the rivet and Equation is modified in the following way:**

$$\boxed{P_s = 2\left[\frac{\pi}{4} d^2 \tau n\right] \quad \text{(for double shear)}}$$



(ii) Tensile Strength of Plate between Rivets: **The tensile failure of the plate between two consecutive rivets in a row is illustrated in Figure below. The width of plate between the two points *A and B is (p – d/2 – d/2) or* (*p – d) and the thickness is t. Therefore, tensile* resistance of the plate between two rivets is given by,**

$$\boxed{P_t = (p - d)\, t\sigma_t}$$

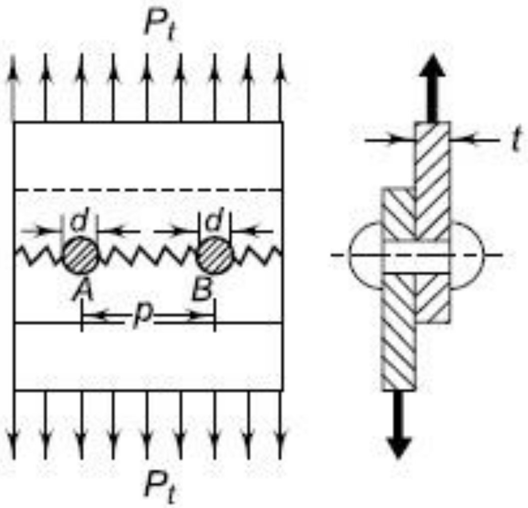
where,

P_t = tensile resistance of plate per pitch length (N)

p = pitch of rivets (mm)

t = thickness of plate (mm)

σ_t = permissible tensile stress of plate material (N/mm^2)



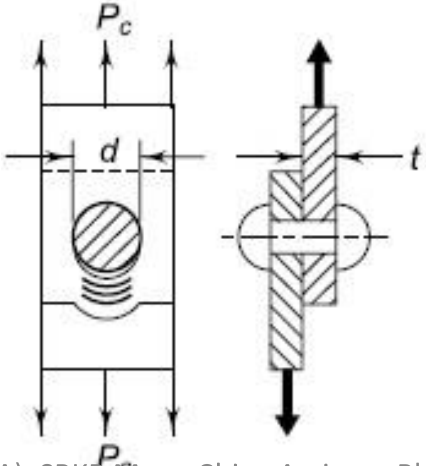
(iii) Crushing Strength of Plate: The crushing **failure of the plate is illustrated in Figure below. This type of failure occurs when the compressive stress between the shank of the rivet and the plate exceeds the yield stress in compression. The failure results in elongating the rivet hole in the plate and loosening of the joint. The crushing resistance of the plate is given by,**

$$\boxed{P_c = dt\sigma_c n}$$

where,

P_c = crushing resistance of plate per pitch length (N)

σ_c = permissible compressive stress of plate material (N/mm^2)



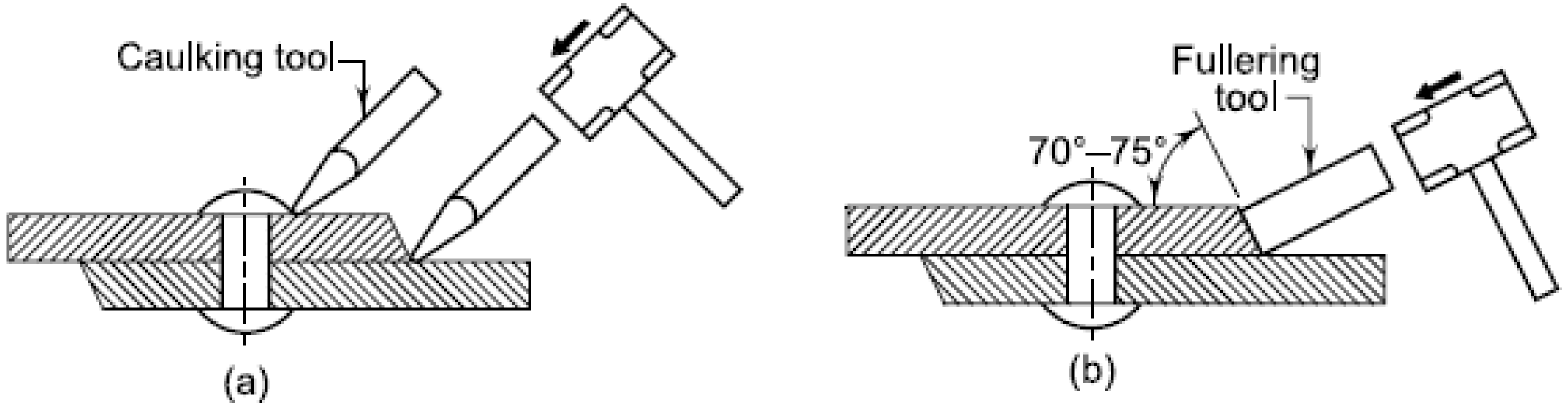
(iv) Efficiency of Joint: **The efficiency of the riveted joint is defined as the ratio of the strength of riveted joint to the strength of unriveted solid plate. The strength of the riveted joint is the lowest value of P_s, P_t and P_c .The strength of solid plate of width, equal to the pitch p and thickness t, subjected to tensile stress σ_t is given by, $P = pt\sigma_t$**

Therefore, the efficiency is given by,

$$\boxed{\eta = \frac{\text{Lowest of } P_s, P_t, \text{and } P_c}{P}}$$

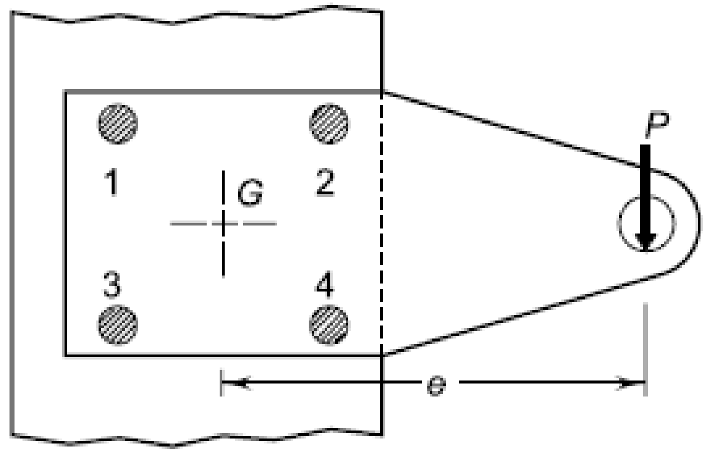
CAULKING AND FULLERING

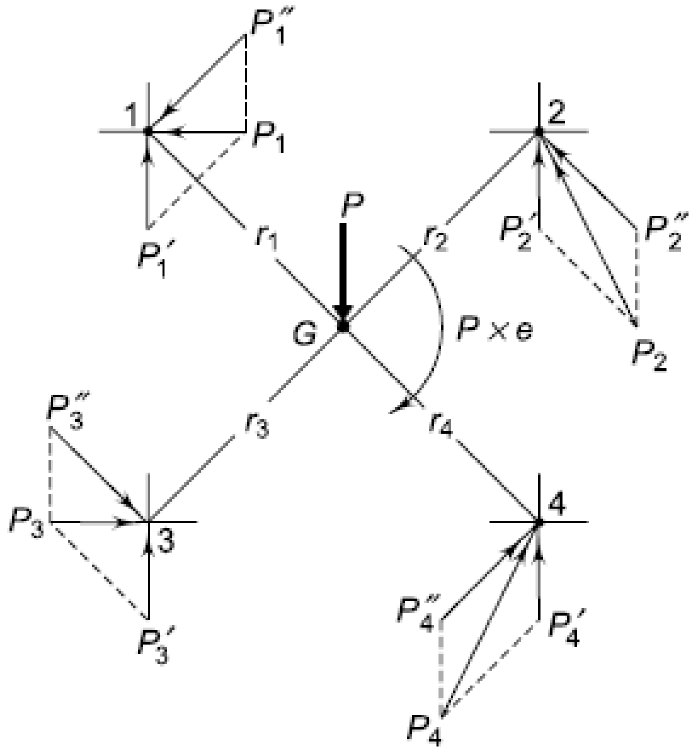
In applications like pressure vessels and boilers, the riveted joint should be leak proof and fluid tight. Caulking and fullering processes are used to obtain such leakproof riveted joints. The caulking process is applied to the edges of plates in a lap joint and the edges of strap plate in a butt joint. Fullering is similar to the caulking process except for the shape of the tool. The width of the fullering tool is equal to the thickness of the plate being hammered.



(a) Caulking operation and (b) Fullering operation

Eccentrically loaded rivets in shear





$$P_1' = P_2' = P_3' = P_4' = \frac{P}{(\text{No. of bolts})}$$

$$P \times e = P_1'' r_1 + P_2'' r_2 + P_3'' r_3 + P_4'' r_4$$

$$P_1'' = C r_1$$
$$P_2'' = C r_2$$
$$P_3'' = C r_3$$
$$P_4'' = C r_4$$

$$C = \frac{Pe}{(r_1^2 + r_2^2 + r_3^2 + r_4^2)}$$

$$P_1'' = \frac{Per_1}{(r_1^2 + r_2^2 + r_3^2 + r_4^2)}$$

$$P_2'' = \frac{Per_2}{(r_1^2 + r_2^2 + r_3^2 + r_4^2)}$$

$$P_2 = \sqrt{(P_2'^2 + P_2''^2 + 2P_2' P_2'' \cos\theta)}$$

$$\tau = \frac{P_2}{A}$$

Design of Springs

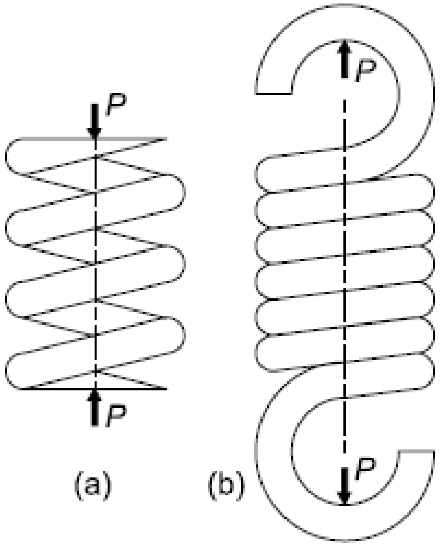
Springs

A spring is defined as an elastic machine element, which deflects under the action of the load and returns to its original shape when the load is removed.

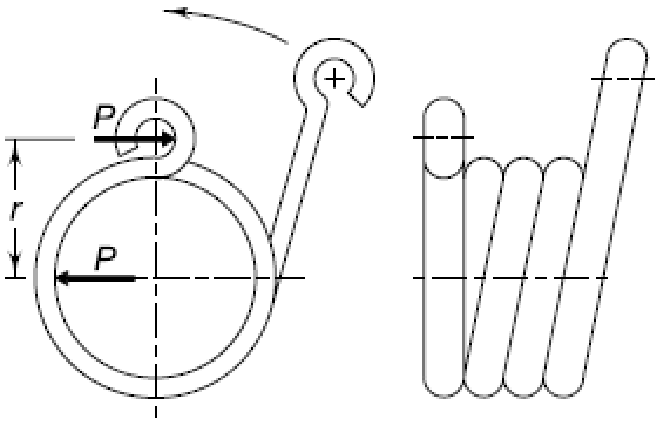
The important functions and applications of springs are as follows:

i. Springs are used to absorb shocks and vibrations, e.g., vehicle suspension springs, railway buffer springs, buffer springs in elevators and vibration mounts for machinery.
ii. Springs are used to store energy, e.g., springs used in clocks, toys, movie-cameras, circuit breakers and starters.
iii. Springs are used to measure force, e.g., springs used in weighing balances and scales.
iv. Springs are used to apply force and control motion. , e.g., In the cam and follower mechanism, spring is used to maintain contact between the two elements, The spring used in clutch provides the required force to engage the clutch.

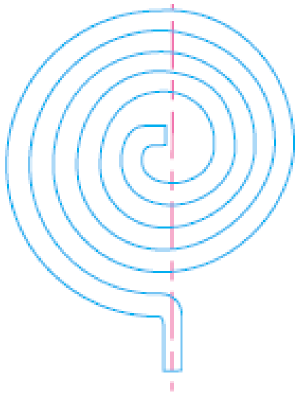
Types of Springs



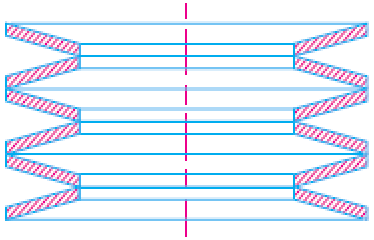
Helical Springs: (a) Compression (b) Extension Spring



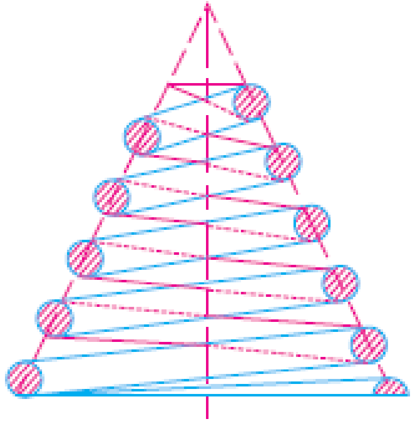
Helical Torsion Spring



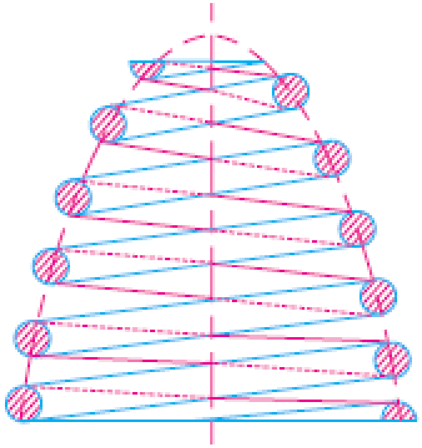
(*b*) Spiral torsion spring.



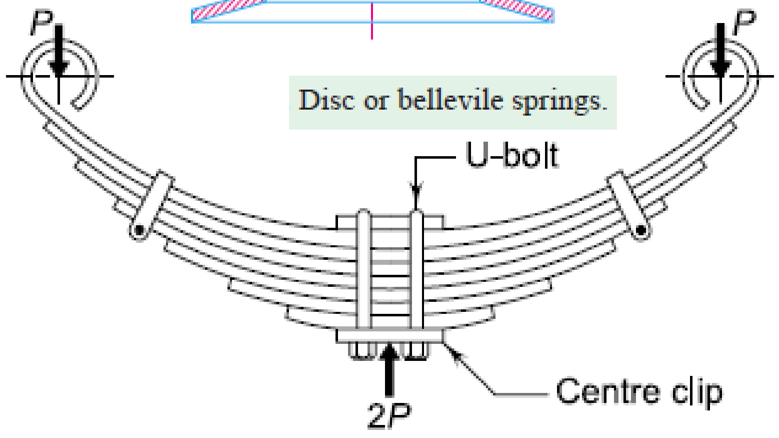
Disc or bellevile springs.



(*a*) Conical spring.



(*b*) Volute spring.



Semi-elliptic Leaf Spring

Springs Materials

The selection of material for the spring wire depends upon the following factors:

i. The load acting on the spring
ii. The range of stress through which the spring operates
iii. The limitations on mass and volume of spring
iv. The expected fatigue life
v. The environmental conditions in which the spring will operate such as temperature and corrosive atmosphere
vi. The severity of deformation encountered while making the spring.

There are four basic varieties of steel wire which are used in springs in the majority of applications:

i. patented and cold-drawn steel wires (unalloyed)
ii. oil-hardened and tempered spring steel wires and valve spring wires
iii. oil-hardened and tempered steel wires (alloyed)
iv. stainless steel spring wires.
v. There are non-ferrous materials, such as spring brass, phosphor bronze, silicon–bronze, monel and beryllium–copper, which are also used in spring wires.

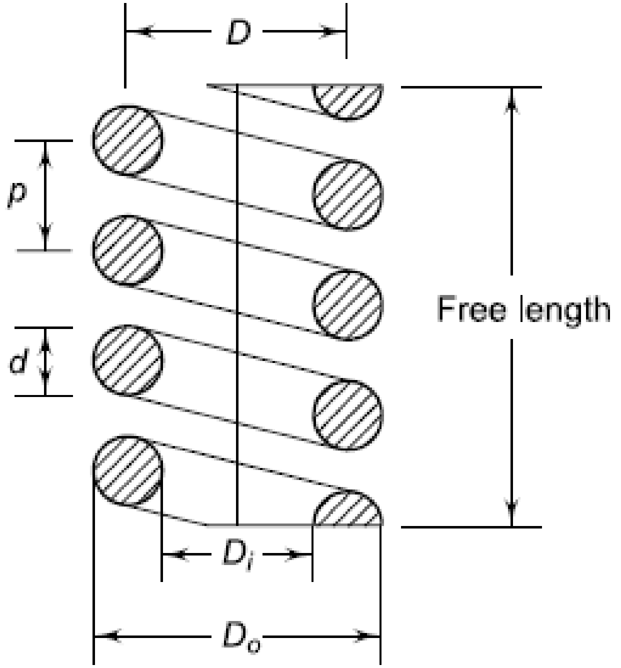
Terminology of Helical Springs

The main dimensions of a helical spring subjected to compressive force are shown in Figure. They are as follows:

d = wire diameter of spring (mm)
D_i = inside diameter of spring coil (mm)
D_o = outside diameter of spring coil (mm)
D = mean coil diameter (mm)

Therefore,

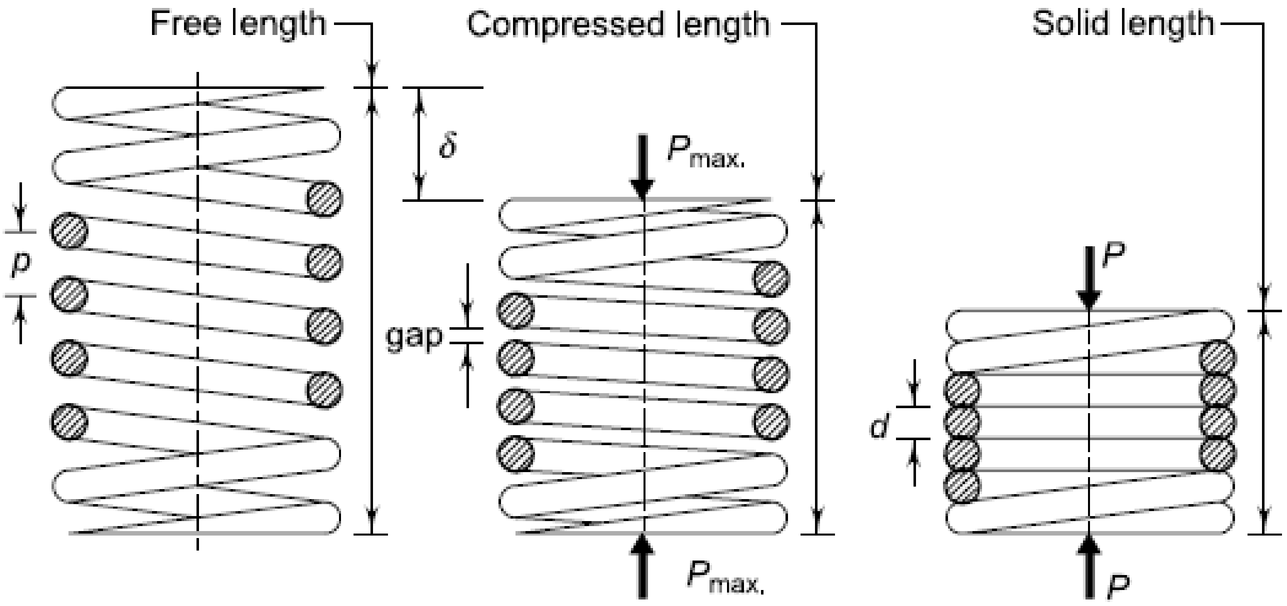
$$D = \frac{D_i + D_o}{2}$$



(a) Spring Index: There is an important parameter in spring design called *spring index*. It is denoted by the letter C. ***The spring index is defined as the ratio of mean coil diameter to wire diameter***.

$$C = \frac{D}{d}$$

(b) *Length of a Spring*: There are three terms—*free length, compressed length and solid length, which are illustrated in* Figure below. These terms are related to helical compression spring. These lengths are determined by the following way:



(a) (b) (c)

(i) Solid Length Solid length is defined as the axial length of the spring which is so compressed that the adjacent coils touch each other. In this case, the spring is completely compressed and no further compression is possible. The solid length is given by,

$$\text{Solid length} = N_t\, d$$

where,

N_t = total number of coils

(ii) Compressed Length: Compressed length is defined as the axial length of the spring, which is subjected to maximum compressive force. In this case, the spring is subjected to maximum deflection δ. When the spring is subjected to maximum force, there should be some gap or clearance between the adjacent coils. The gap is essential to prevent clashing of the coils. The clashing allowance or the total axial gap is usually taken as 15% of the maximum deflection. Sometimes, an arbitrary decision is taken and it is assumed that there is a gap of 1 or 2 mm between adjacent coils under maximum load condition. In this case, the total axial gap is given by,

$$\text{Total gap} = (N_t - 1) \times \text{Gap between adjacent coils}$$

(iii) Free Length: Free length is defined as the axial length of an unloaded helical compression spring. In this case, no external force acts on the spring. Free length is an important dimension in spring design and manufacture. It is the length of the spring in free condition prior to assembly. Free length is given by,

$$\begin{aligned} \text{free length} &= \text{compressed length} + \delta \\ &= \text{solid length} + \text{total axial gap} + \delta \end{aligned}$$

(c) Pitch of the coil: The *pitch of the coil* is defined as the axial distance between adjacent coils in uncompressed state of spring. It is denoted by *p*. It is given by,

$$p = \frac{\text{free length}}{(N_t - 1)}$$

(d) Stiffness of the spring: The *stiffness of the spring* (*k*) is defined as the force required to produce unit deflection. There are various names for stiffness of spring such as *rate of spring*, *gradient of spring*, *scale of spring* or *simply spring constant*.

$$k = \frac{P}{\delta}$$

(e)Active coils are the coils in the spring which contribute to spring action, support the external force and deflect under the action of force.

(f) Inactive Coils: A portion of the end coils, which is in contact with the seat, does not contribute to spring action and are called *inactive coils*. These coils do not support the load and do not deflect under the action of an external force. The number of inactive coils is given by,

$$\text{inactive coils} = N_t - N$$

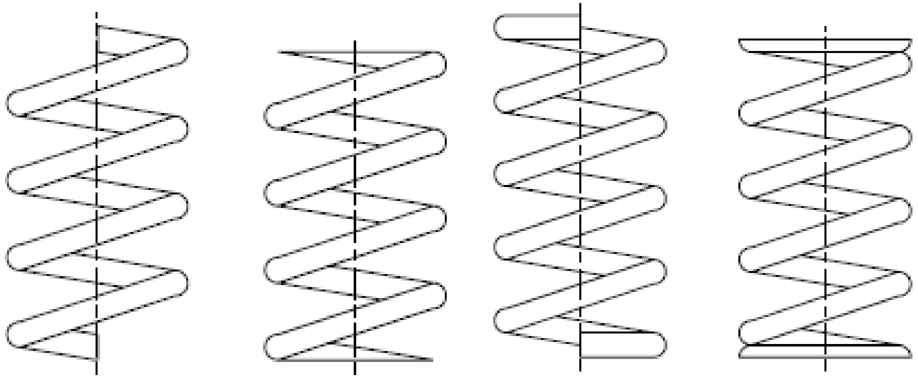
where,

N = number of active coils.

Styles of Ends

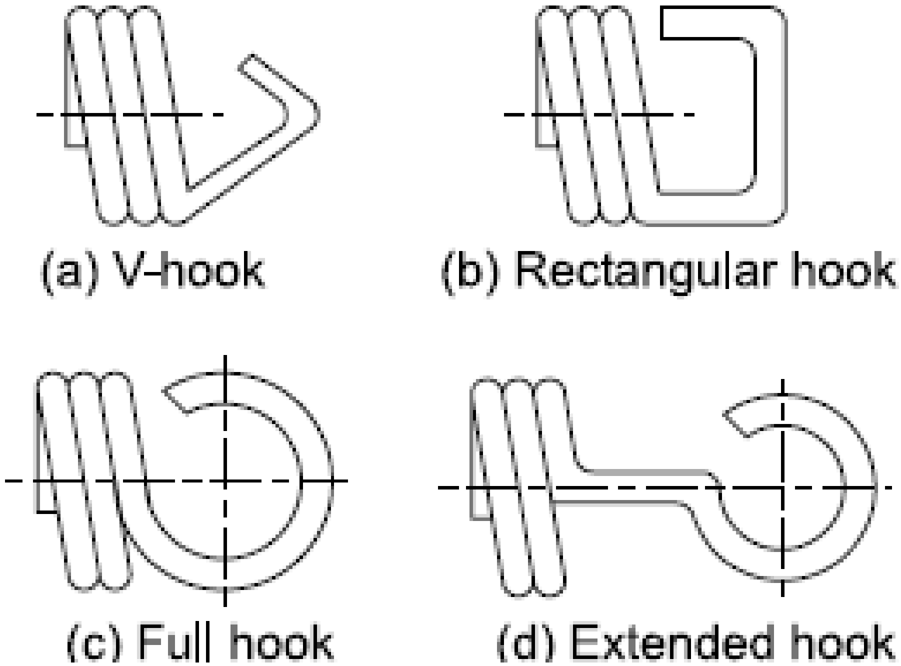
- There are four common methods which are used in forming the ends of the helical compression spring as shown in Figure below-***plain ends, plain and ground ends, square ends and square and ground ends***.
- The turns at the two ends do not affect the deflection calculated by the load-deflection equation.
- Therefore, while calculating the number of active turns, the end turns should be subtracted from the total number of turns.
- The number of active turns for different styles of end is as follows:

Type of ends	*Number of active turns (N)*
Plain ends	N_t
Plain ends (ground)	$\left(N_t - \frac{1}{2}\right)$
Square ends	$(N_t - 2)$
Square ends (ground)	$(N_t - 2)$

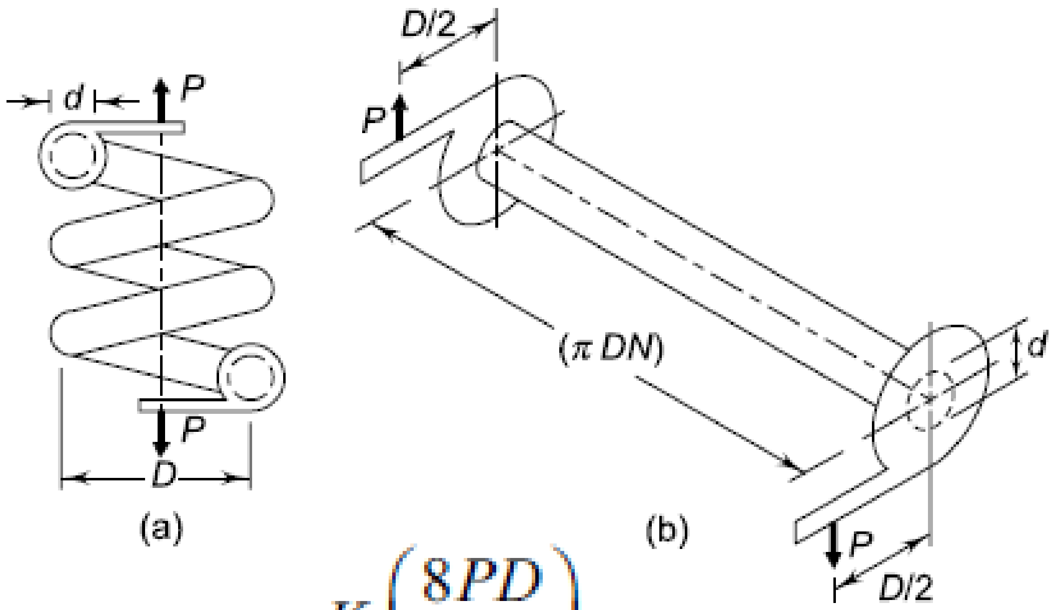


(a) Plain ends (b) Plain and ground ends (c) Square ends (d) Square and ground ends

- The different styles of end for the helical extension spring are shown in Figure below.
- The end should be designed in such a way that the stress concentration at the bend is minimum.
- Sometimes the effect of stress concentration in ends is so severe that the spring body becomes stronger than the end and failure occurs in the end coils.
- For helical extension ends, all coils are active coils. The number of active coils (*N*) is the same as the total number of coils (*Nt*).



Stress and Deflection Equations



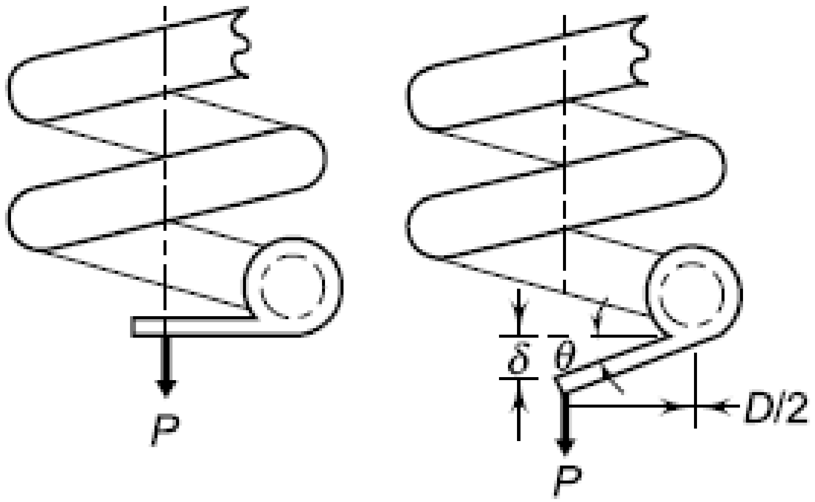
$$\tau = K\left(\frac{8PD}{\pi d^3}\right)$$

where K is called the *stress factor* or *Wahl factor*.

The Wahl factor is given by,

$$K = \frac{4C-1}{4C-4} + \frac{0.615}{C}$$

where C is the spring index.



$$\delta = \theta \times (\text{length of bracket}) \\ = \theta \times (D/2)$$

$$\delta = \frac{8PD^3N}{Gd^4}$$

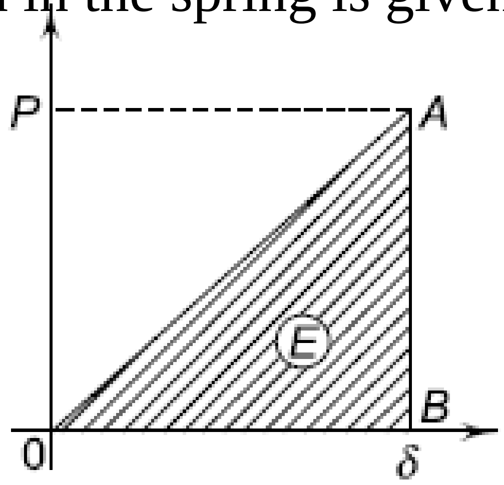
- The load is linearly proportional to the deflection of the spring. The load-deflection curve for helical spring is shown in Figure below.
- The area below the load-deflection line gives the strain energy stored in the spring. Assuming that the load is gradually applied, the energy stored in the spring is given by,

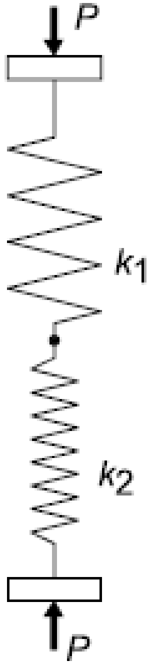
$$E = \text{area below load-deflection line}$$

$$= \text{area of triangle } OAB = \frac{1}{2}\overline{OB} \times \overline{BA} = \frac{1}{2}\delta P$$

$$E = \frac{1}{2} P\,\delta \quad \text{where,}$$

E = strain energy stored in spring (N-mm)



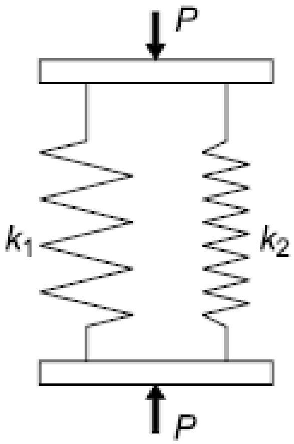


For series connection,

(i) The force acting on each spring is same and equal to the external force

(ii) The total deflection of the spring combination is equal to the sum of the deflections of individual springs

$$\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2} + \cdots$$



For parallel connection,

(i) The force acting on the spring combination is equal to the sum of forces acting on individual springs

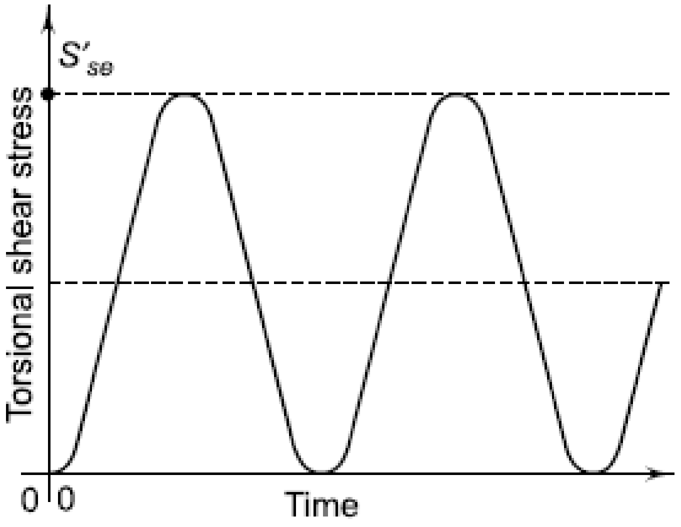
(ii) The deflection of individual springs is same and equal to the deflection of the combination

$$k = k_1 + k_2 + \cdots$$

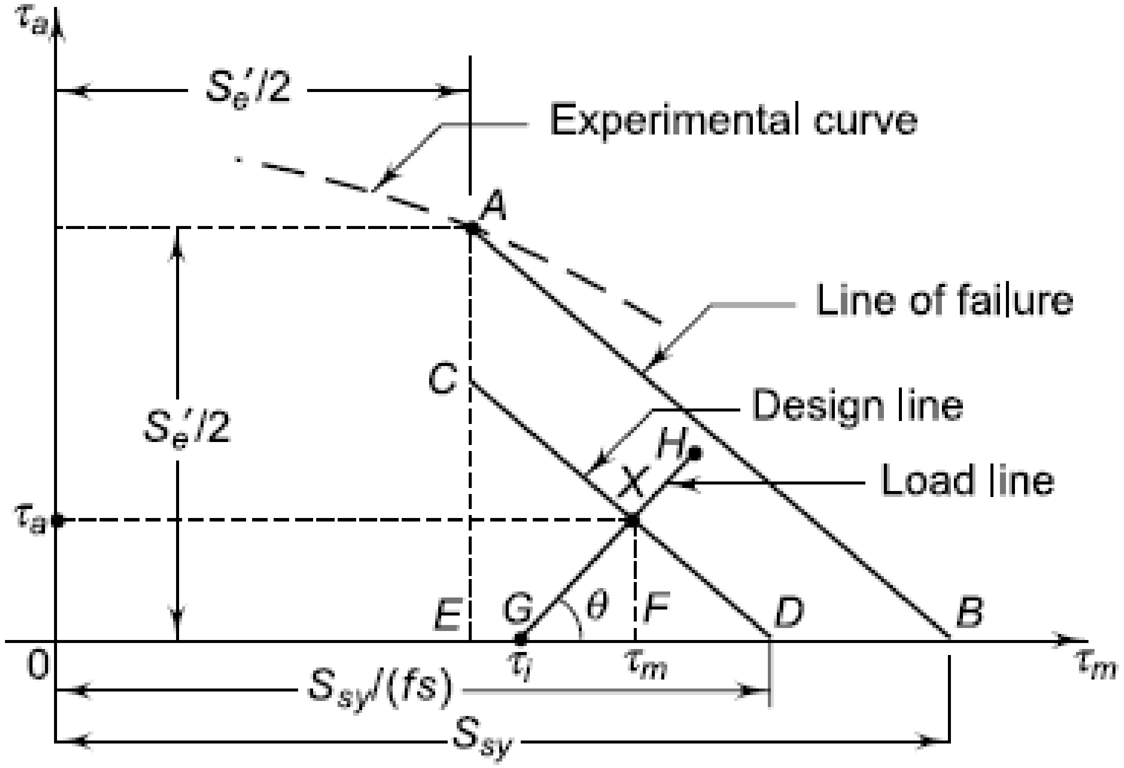
Design Against Fluctuating Load

$$P_m = \frac{1}{2}(P_{max.} + P_{min.}) \qquad \tau_m = K_s\left(\frac{8P_mD}{\pi d^3}\right) \qquad \tau_a = K\left(\frac{8P_aD}{\pi d^3}\right)$$

$$P_a = \frac{1}{2}(P_{max.} - P_{min.}) \qquad K_s = \left(1 + \frac{0.5}{C}\right) \qquad K = \frac{4C-1}{4C-4} + \frac{0.615}{C}$$



$$\frac{\tau_a}{\frac{S_{sy}}{(fs)} - \tau_m} = \frac{\frac{1}{2}S'_{se}}{S_{sy} - \frac{1}{2}S'_{se}}$$



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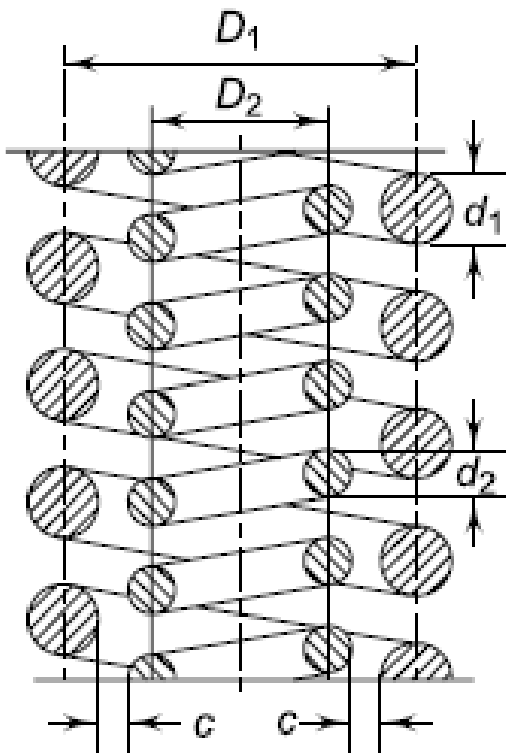
Concentric Springs

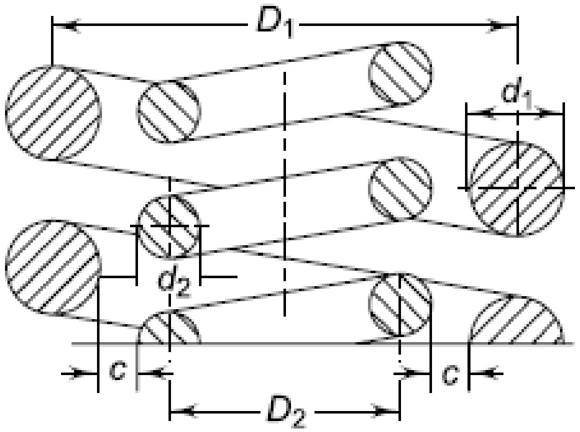
- A concentric spring consists of two helical compression springs, one inside the other, having the same axis. Concentric spring is also called a ‘*nested*’ spring.
- Concentric spring has the following advantages:
  i. Since there are two springs, the load carrying capacity is increased and heavy load can be transmitted in a restricted space.
  ii. In concentric spring, the operation of the mechanism continues even if one of the springs breaks. This results in ‘*fail safe*’ system.
  iii. In concentric spring, the spring vibrations called ‘*surge*’, are eliminated.
- The design analysis of concentric spring is based on the following assumptions:
  i. The springs are made of the same material.
  ii. The maximum torsional shear stresses induced in outer and inner springs are equal.
  iii. They have the same free length.
  iv. Both springs are deflected by the same amount and therefore, have same solid length.

d_1 = wire diameter of outer spring
d_2 = wire diameter of inner spring
D_1 = mean coil diameter of outer spring
D_2 = mean coil diameter of inner spring
P_1 = axial force transmitted by outer spring
P_2 = axial force transmitted by inner spring
P = total axial force
δ_1 = deflection of outer spring
δ_2 = deflection of inner spring
N_1 = number of active coils in outer spring
N_2 = number of active coils in inner spring



$$\frac{P_1}{P_2} = \frac{d_1^2}{d_2^2}$$

$$\frac{P_1}{P_2} = \frac{\pi d_1^2}{\pi d_2^2} = \frac{a_1}{a_2}$$



$$\frac{d_1}{d_2} = \frac{C}{(C-2)}$$

Surge in Spring

- When the natural frequency of vibrations of the spring coincides with the frequency of external periodic force, which acts on it, resonance occurs.
- In this state, the spring is subjected to a wave of successive compressions of coils that travels from one end to the other and back. This type of vibratory motion is called ‘*surge*’ of spring. Surge is found in valve springs, which are subjected to periodic force.
- Let us consider a helical compression spring, one end of which is held against a flat rigid surface and the other end is subjected to a periodic external force. It always takes some ‘time’ to transmit this force from one end to the other.
- The force is transmitted by compression of coils. Initially, the end coil in contact with external force compresses and then it transmits a large portion of its compression to the adjacent second coil. The second coil compresses and in turn, transmits its major compression to the third coil.

- This process continues and a wave of compressed coils travels along the length of the spring till it reaches the end supported on rigid surface where it is ‘reflected’ back. The reflected wave travels back to the end in contact with the external force.
- If the time required for the wave to travel from one end to the other and back coincides with the time interval between successive load applications of periodic external force, resonance occurs and very large deflections of the coils are produced resulting in very high stresses.
- Many times, this stress in the coils is more than the endurance limit stress of the spring and fatigue failure occurs. Surge is the main cause of failure in valve springs.
- The natural frequency of helical compression springs held between two parallel plates is given by,

$$\omega = \frac{1}{2}\sqrt{\frac{k}{m}}$$

Surge in springs is avoided by the following methods:

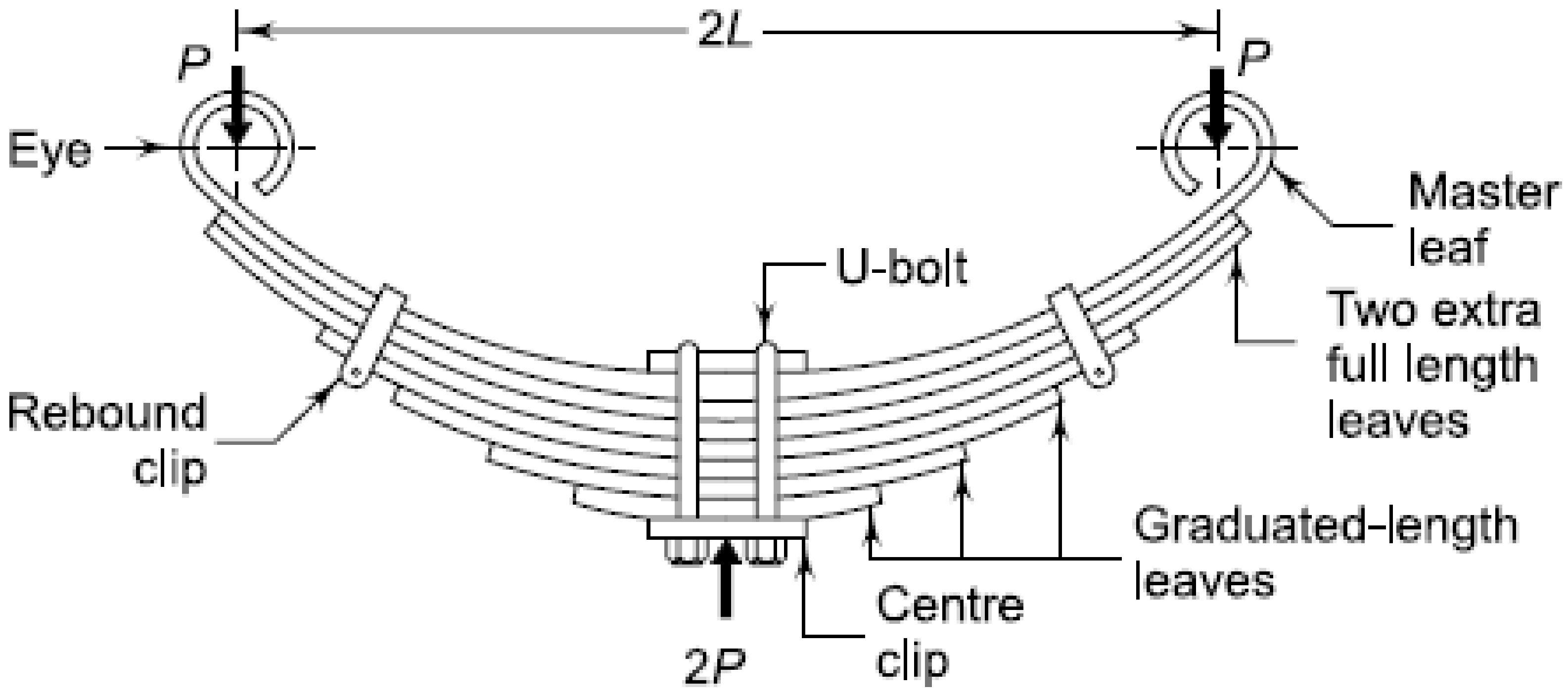
i. The spring is designed in such a way that the natural frequency of the spring is 15 to 20 times the frequency of excitation of the external force. This prevents the resonance condition to occur.
ii. The spring is provided with friction dampers on central coils. This prevents propagation of surge wave.
iii. A spring made of stranded wire reduces the surge. In this case, the wire of the spring is made of three strands. The direction of winding of strands is opposite to the direction of winding of the coils while forming the spring. In case of compression of the coils, the spring tends to wind the individual wires closer together, which introduces friction. This frictional damping reduces the possibility of surge.

Shot Peening

- ❖ In a large number of applications, the external force acting on the spring fluctuates with respect to time resulting in fatigue failure.
- ❖ Due to poor surface finish of the spring wire, the fatigue crack usually begins with some surface irregularity and propagates due to tensile stresses.
- ❖ It has been observed that propagation of fatigue crack is always due to tensile stresses.
- ❖ In order to reduce the chances of crack propagation, a layer of residual compressive stress is induced in the surface of the spring wire.
- ❖ One of the methods of creating such a layer is shot peening.

- In this process, small steel balls are impinged on the wire surface with high velocities either by an air blast or by centrifugal action.
- The balls strike against the wire surface and induce residual compressive stresses.
- The depth of the layer of the residual compressive stresses depends upon a number of factors, such as size of the balls, velocity of striking, original hardness and ductility of the spring wire.
- These parameters are adjusted in such a manner as to produce the required depth of layer of compressive stress.
- Shot peening is effective for springs loaded only in one direction, such as helical compression, helical extension or torsion bar springs.

Multi-Leaf Spring



n_f = number of extra full-length leaves

n_g = number of graduated-length leaves including master leaf

n = total number of leaves

b = width of each leaf (mm)

t = thickness of each leaf (mm)

L = length of the cantilever or half the length of semi-elliptic spring (mm)

P = force applied at the end of the spring (N)

P_f = portion of P taken by the extra full-length leaves (N)

P_g = portion of P taken by the graduated-length leaves (N)

- Multi-leaf springs are widely used for the suspension of cars, trucks and railway wagons.
- A multi-leaf spring consists of a series of flat plates, usually of semi-elliptical shape. The flat plates are called *leaves* of the spring. The leaves have graduated lengths. The leaf at the top has maximum length. The length gradually decreases from the top leaf to the bottom leaf. The longest leaf at the top is called *master leaf*. It is bent at both ends to from the spring eyes.
- Two bolts are inserted through these eyes to fix the leaf spring to the automobile body. The leaves are held together by means of two U-bolts and a centre clip. Rebound clips are provided to keep the leaves in alignment and prevent lateral shifting of the leaves during operation. At the centre, the leaf spring is supported on the axle.
- Multi-leaf springs are provided with one or two extra full length leaves in addition to master leaf. The extra full-length leaves are stacked between the master leaf and the graduated length leaves. The extra full-length leaves are provided to support the transverse shear force.

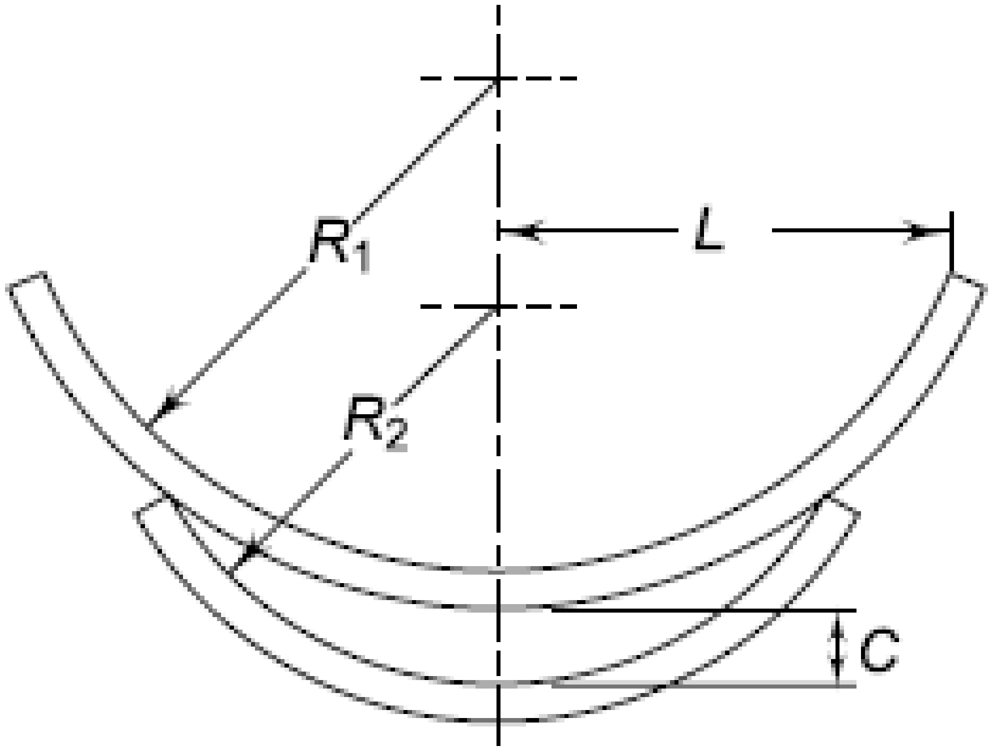
- For the purpose of analysis, the leaves are divided into two groups namely, master leaf along with graduated-length leaves forming one group and extra full-length leaves forming the other.
- Multi-leaf springs are designed using load-stress and load-deflection equations.

$$\sigma_b = \frac{6PL}{nbt^2} \qquad \delta = \frac{12PL^3}{Ebt^3(3n_f + 2n_g)}$$

- The standard dimensions for the width and thickness of the leaf section are as follows:
  - *Nominal thickness (mm)*: 3.2, 4.5, 5, 6, 6.5, 7, 7.5, 8, 9, 10, 11, 12, 14, and 16
  - *Nominal width (mm):* 32, 40, 45, 50, 55, 60, 65, 70, 75, 80, 90, 100 and 125
- The leaves are usually made of steels, 55Si2Mn90, 50Cr1 or 50CrlV23. The plates are hardened and tempered.
- The factor of safety based on the yield strength is from 2 to 2.5 for the automobile suspension.

Nipping of Leaf Springs

- In general, the stresses in extra full-length leaves are 50% more than the stresses in graduated-length leaves. One of the methods of equalising the stresses in different leaves is to pre-stress the spring.
- The pre-stressing is achieved by bending the leaves to different radii of curvature, before they are assembled with the centre clip. The full-length leaf is given a greater radius of curvature than the adjacent leaf. The radius of curvature decreases with shorter leaves.
- The initial gap *C* between the extra full-length leaf and the graduated-length leaf before the assembly, is called a '*nip*'.
- Such pre-stressing, achieved by a difference in radii of curvature, is known as '*nipping*'. Nipping is common in automobile suspension springs.



$$C = \frac{2PL^3}{Enbt^3}$$

The initial pre-load *Pi* required to close the gap *C* between the extra full-length leaves and graduated length leaves

$$P_i = \frac{2n_g n_f P}{n(3n_f + 2n_g)}$$